Nonlinear observer-based control for an active rectifier

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In this paper, a nonlinear observer-based controller for a single-phase active front-end (AFE) rectifier is presented to improve power factor with high quality output DC voltage. Proposed algorithm is based on an Luenberger-like observer and a passivity-based controller. Closed-loop stability is demonstrated for all operating ranges using Lyapunov theory. Performance of observer-based controller is shown by simulation for voltage sag occurrence and abrupt load changes.

Keywords: Active front-end (AFE) rectifier, Nonlinear observer-based controller, Power factor

Introduction

Power quality, a very important issue of power delivery1, is measured by power factor (PF), which of an AC electric power system is the ratio of real power and apparent power; an unity-PF assures the best use of energy. In spite of PF is decreased by nonlinear loads, they are frequently used on industry applications. One of the most common nonlinear loads is passive-front end (PFE) rectifier, which is based on a diode bridge rectifier followed by a large capacitor2. Advantages of using PFE rectifiers are low cost, simple structure, robustness, and absence of control policy. But a PFE rectifier gives a low PF and DC voltage ($V_{bus}$) reduces due to a voltage sag3. These problems can be solved by using active front-end (AFE) rectifiers, which improve PF, reduce harmonic content on distorted current, and give high quality output DC voltage. There exists several works to cope with this problem. A hysteresis based current control4-6 is developed to achieve high PF and low current harmonic distortion. Two methodologies based on D-Q reference-frame transformation are discussed7; the first one uses a sine-cosine composition, while the other is based on a bi-phase transformation. A master and slave control-loops are proposed8, outer control-loop (slave loop) has a proportional-integral (PI) voltage compensator that is designed to accomplish DC-bus voltage regulation. In master control loop, a carrier-based current-control to track line reference current is proposed. An active power filter9 to achieve a high low order harmonics reduction is proposed. A pulse-width modulation (PWM) switching pattern, which allows an adequate DC voltage regulation in steady state, is presented10. A carrier-based current control and a PI voltage compensator are proposed to track line current and regulate DC-link voltage, respectively11. A multilevel indirect field oriented control applied to a motor drive gives a good performance12 in high and low speed.

Since AFE rectifier has a highly nonlinear behavior, classical control techniques are not the most suitable choice to design a high performance controller for this kind of systems. Besides, the use of nonlinear control techniques allows to show the closed-loop system stability over all operating points. Furthermore, sometimes it is useful to eliminate some sensors to reduce costs using state observers.

This study presents a nonlinear observer-based controller for a single-phase AFE rectifier to give unity-PF and high quality output DC voltage. Proposed algorithm is based on an Luenberger-like observer and a passivity-based controller (PBC), besides closed-loop system stability using Lyapunov theory.
Experimental Section

Single-Phase AFE Rectifier: Principle of Operation

Power topology of single-phase AFE rectifier used in this study is composed by an H-bridge noncontrolled diode rectifier, an inductor filter \((L_s)\), two power switches \(S_1\) and \(S_2\), and two split capacitors as DC bus (Fig. 1).

Rectifier provides two balanced DC voltages. Power switches generate a sinusoidal-controlled voltage between \(a\) and \(b\) nodes \((v_{ab})\) to obtain a sinusoidal current with very low harmonic content and high power factor. This topology has several advantages: i) It operates as boost type rectifier allowing power factor correction; ii) A controlled voltage generated between \(a\) and \(b\) nodes allows to guarantee a low ripple in reference tracking tasks; and iii) It is possible to control power flow from AC side to DC side to check on voltage sag. Detailed principle of operation and a complete analysis in steady state of this topology is reported\(^4\).

Assuming that \(S_1\) and \(S_2\) have same switching pattern, a three-level PWM waveform and four operation modes of rectifier are achieved. So, applying Kirchoff laws to power converter, state-space model representation of single-phase AFE rectifier is given as

\[
\begin{align*}
\dot{x}_1 &= -\frac{R_s}{L_s} x_1 - \frac{\text{sign}(v_{ab})}{L_s} (1-u) (x_2 + x_3) + \frac{v_s}{L_s} \\
\dot{x}_2 &= \text{sign}(v_{ab}) (1-u) \frac{x_1}{C_1} - \frac{x_2}{R_s(t) C_1} \\
\dot{x}_3 &= \text{sign}(v_{ab}) (1-u) \frac{x_1}{C_2} - \frac{x_3}{R_s(t) C_2}
\end{align*}
\]

where

- \(x_1\) : Inductor current.
- \(x_2, x_3\) : DC bus capacitors voltages.
- \(L_s\) : Inductor.
- \(C_1, C_2\) : DC bus capacitors.
- \(R_s\) : Associated inductor resistance.
- \(R_s(t), R_s(t)\) : Linear or nonlinear loads.
- \(u\) : Continuous control signal for \(S_1\) and \(S_2\) represented as duty cycle, \(0 < u < 1\).
- \(v_s\) : AC mains voltage.

Observability and Observer Design

Nonlinear Observability

First, a necessary condition for observer existence is given by a structural property called system’s observability, which implies the possibility of estimating state variables of the system from inputs and outputs. In nonlinear systems, a systematic solution to this problem does not exist, because observability of nonlinear systems depend on applied input. However, some results related with observability for specific non-linear systems exist.

Consider a nonlinear time-varying system as

\[
\begin{align*}
\dot{x} &= f(x, u, t), \; t > t_0, \; x(t_0) = x_0 \\
y &= h(x, u, t) \quad \ldots (2)
\end{align*}
\]

where state \(x \in \mathbb{R}^n\), input \(u \in \mathbb{R}^p\) and output \(y \in \mathbb{R}^q\). Vector field \(f(x, u, t)\) and function \(h(x, u, t)\) are assumed to be real, analytic and sufficiently smooth.

System (2) is called locally observable if \((nq \times n)\) observability matrix

\[
Q(x, \hat{u}, t) = \begin{bmatrix}
\frac{\partial h}{\partial x} & \psi f \frac{\partial h}{\partial x} & \ldots & \psi f^{n-1} f \frac{\partial h}{\partial x}
\end{bmatrix}^T \quad \ldots (3)
\]

has full rank in considered domains of \(x, u\) and \(t\), where

\[
u = \begin{bmatrix}
u, u, \ldots, u \end{bmatrix}^T \quad \text{and linear differential operator}
\]

\[
\psi f \text{ is } \psi f \frac{\partial h}{\partial x} = \begin{bmatrix}
\psi f \frac{\partial h_1}{\partial x} & \ldots & \psi f \frac{\partial h_q}{\partial x}
\end{bmatrix}^T \text{ with}
\]

\[
\psi f \frac{\partial h_i}{\partial x} = \frac{\partial}{\partial t} \left( f \frac{\partial h_i}{\partial x} \right) + u^T \left[ \frac{\partial}{\partial u} \left( f \frac{\partial h_i}{\partial x} \right) \right]^T + \frac{\partial h_i}{\partial x} f^T \frac{\partial}{\partial x} \left( f \frac{\partial h_i}{\partial x} \right), \quad i \in \{1, \ldots, q\}
\]
Observer in canonical form depends on selected linearly independent rows of observability matrix (3), that are needed to determinate transformation. So, selected rows are sorted in a \( (n \times n) \) matrix \( S \) as

\[
Q_s(x,u,t) = \begin{bmatrix}
\frac{dh_1}{dx} & \psi f \frac{dh_1}{dx} & \cdots & \psi^{n-1} f \frac{dh_1}{dx} & \frac{dh_2}{dx} & \psi f \frac{dh_2}{dx} & \cdots & \psi^{n-1} f \frac{dh_2}{dx} \\
\end{bmatrix}^T \tag{4}
\]

where \( \psi = \sum_{i=1}^{n} n_i = n \)

This matrix must have full rank in the considered domains of \( x, u \) and \(< t_{13,14} \).

**Single-Phase AFE Rectifier Observability**

The system observability can be demonstrated taking \( y = [h_1 \ h_2]^T = [x_2 \ x_3]^T \) and considering observability matrix \( Q_s(x,u,t) \) as

\[
Q_s(x,u,t) = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-\text{sign}(v_s)(1-u) & \frac{R_i(t)}{C_1} & 0 \\
\end{bmatrix}
\]

which results in a full range matrix and then, the system is locally observable.

**Observer Design**

Once observability of AFE rectifier has been verified, a Luenberger-like observer to estimate \( x_1 \) using measured variables \( x_2 \) and \( x_3 \) can be proposed as

\[
\begin{align*}
\dot{x}_1 &= -\frac{R_s}{L_s} \dot{x}_1 - \text{sign}(v_s)(1-u) (\dot{x}_2 + \dot{x}_3) + \frac{v_s}{L_s} + \frac{k_{11}}{L_s} e_1 + \frac{k_{31}}{L_s} e_3 \\
\dot{x}_2 &= \text{sign}(v_s)(1-u) \frac{\dot{x}_2}{C_1} + \frac{k_{21}}{C_1} e_2 + \frac{k_{32}}{C_1} e_3 \\
\dot{x}_3 &= \text{sign}(v_s)(1-u) \frac{\dot{x}_3}{C_2} + \frac{k_{31}}{C_2} e_1 + \frac{k_{32}}{C_2} e_2
\end{align*}
\]

where \( k_{11}, k_{12}, k_{22}, k_{31}, k_{32} \) are observer gains.

Defining estimation errors \( e_1 = x_1 - \hat{x}_1, \)

\( e_2 = x_2 - \hat{x}_2, \) and \( e_3 = x_3 - \hat{x}_3, \) observation error dynamics are given as

\[
\begin{align*}
\dot{e}_1 &= -\frac{R_s}{L_s} e_1 - \text{sign}(v_s) (1-u) (e_2 + e_3) - \frac{k_{11}}{L_s} e_1 - \frac{k_{31}}{L_s} e_3 \\
\dot{e}_2 &= \text{sign}(v_s)(1-u) \frac{e_2}{C_1} + \frac{k_{21}}{C_1} e_2 - \frac{k_{22}}{C_1} e_3 \\
\dot{e}_3 &= \text{sign}(v_s)(1-u) \frac{e_3}{C_2} - \frac{k_{31}}{C_2} e_2 - \frac{k_{32}}{C_2} e_3
\end{align*}
\]

Observer convergence is proved by Lyapunov function as

\[
V(e) = \frac{1}{2} [L_s e_1^2 + C_1 e_2^2 + C_2 e_3^2]
\]

whose derivative along the trajectories of observation error dynamics (7) is given as

\[
\dot{V}(e) = L_s e_1 \dot{e}_1 + C_1 e_2 \dot{e}_2 + C_2 e_3 \dot{e}_3
\]

\[
\dot{V}(e) = -R_s e_1^2 - \text{sign}(v_s)(1-u) (e_2 + e_3) - \frac{1}{R_1} e_1^2 - \frac{k_{22}}{C_1} e_2^2
\]

\[
\dot{V}(e) = -R_s e_1^2 - \text{sign}(v_s)(1-u) e_3 - \frac{1}{R_2} e_2^2 - \frac{1}{R_3} e_3^2
\]

\[
\dot{V}(e) = \frac{1}{R_2} + k_{32} \]

which results in a definite negative as
\[
V_2(\varepsilon) = \left[ \sqrt{R_2 - \alpha_1} \varepsilon_1 + \sqrt{\left( \frac{1}{R_1} + k_2 - \alpha_2 \right)} \varepsilon_2 + \sqrt{\left( \frac{1}{R_1} + k_2 - \alpha_3 \right)} \varepsilon_3 \right] ^2 \\
- \alpha_1 \varepsilon_1 - \alpha_2 \varepsilon_2 - \alpha_3 \varepsilon_3 < 0, \quad \forall \varepsilon_i \neq 0, \forall \varepsilon_i \neq 0 \text{ and } \forall \varepsilon_i \neq 0 \\
\ldots(10)
\]

and asymptotically stability of observation error dynamics can be ensured.

**Controller Design**

**Controller Design with Measured Variables**

A PBC was selected due to its design and analysis procedure uses an input-output description based on energy considerations. Moreover, in AFE rectifier, it is easy to relate state variables to energy quantities. PBC has been applied successfully in other kinds of power converters. Since the model of AFE rectifier is strictly passive, relative degree as well as minimum phase conditions are satisfied, then a PBC can be easily designed. Defining the desired system as

\[
\begin{align*}
\dot{x}_1^* &= - \frac{R_s}{L_s} x_1^* - \frac{\text{sign}(v_s)}{L_s} (1-u) (x_2^* + x_1^*) + \frac{v_s}{L_s} + \frac{R_p}{L_s} e_{sl} \\
\dot{x}_2^* &= \text{sign}(v_s) (1-u) \frac{x_1^*}{C_1} - \frac{x_2^*}{R_1 C_1} \\
\dot{x}_3^* &= \text{sign}(v_s) (1-u) \frac{x_1^*}{C_2} - \frac{x_3^*}{R_2 C_2} \ldots(11)
\end{align*}
\]

where \(R_p\) is a damping injection term, \(x_1^*, x_2^*, \) and \(x_3^*\) are desired state variables and \(e_{sl} = x_i - x_i^*\) is tracking error. Then, tracking error dynamics are described as

\[
\begin{align*}
\dot{e}_{s1} &= - \frac{R_s}{L_s} e_{s1} - \frac{\text{sign}(v_s)}{L_s} (1-u) (e_{s2} + e_{s3}) - \frac{R_p}{L_s} e_{s1} \\
\dot{e}_{s2} &= \text{sign}(v_s) (1-u) \frac{e_{s1}}{C_1} - \frac{e_{s2}}{R_1 C_1} \\
\dot{e}_{s3} &= \text{sign}(v_s) (1-u) \frac{e_{s1}}{C_2} - \frac{e_{s3}}{R_2 C_2} \ldots(12)
\end{align*}
\]

with \(e_{s2} = x_2 - x_2^*\) and \(e_{s3} = x_3 - x_3^*\).

Using Lyapunov function candidate

\[
V(e_s) = \frac{1}{2} [L_s e_{s1}^2 + C_1 e_{s2}^2 + C_2 e_{s3}^2] , \quad \text{whose derivative along trajectories of (12) is}
\]

\[
\begin{align*}
V(e_s) &= L_s e_{s1} \dot{e}_{s1} + C_1 e_{s2} \dot{e}_{s2} + C_2 e_{s3} \dot{e}_{s3} \\
V(e_s) &= -R_s e_{s1} - \text{sign}(v_s) (1-u) (e_{s2} + e_{s3}) e_{s1} - R_p e_{s1} \\
&\quad + \text{sign}(v_s) (1-u) e_{s2} - \frac{\dot{e}_{s1}}{R_1} + \text{sign}(v_s) (1-u) e_{s3} - \frac{\dot{e}_{s1}}{R_2} \\
V(e_s) &= -[R_s + R_p] e_{s1} - \frac{\dot{e}_{s1}}{R_1} - \frac{\dot{e}_{s1}}{R_2} < 0, \forall e_{s1} \neq 0, \forall e_{s2} \neq 0 \text{ and } \forall e_{s3} \neq 0 \\
\ldots(10)
\end{align*}
\]

Asymptotically stability of closed-loop system can be achieved taking \(R_p > R_s\). Then \(x_1 \rightarrow x_1^*, x_2 \rightarrow x_2^*\) and \(x_3 \rightarrow x_3^*\) as \(t \rightarrow \infty\). Now, to obtain control policy \(u\), desired dynamic (11) of AFE rectifier has to be solved. Renaming \(x_1^*, x_2^*, \) and \(x_3^*\) as \(\zeta_1, \zeta_2, \) respectively, controller dynamic is given as

\[
u = \text{sign}(v_s) \frac{L_s x_1^* + R_s x_1^* - v_s - R_p (x_1 - x_1^*)}{(\zeta_1 + \zeta_2)} + 1
\]

\[\zeta_1 = \text{sign}(v_s) (1-u) \frac{x_1^*}{C_1} - \frac{\zeta_1}{R_1 C_1} \]

\[\zeta_2 = \text{sign}(v_s) (1-u) \frac{x_1^*}{C_2} - \frac{\zeta_2}{R_2 C_2} \ldots(13)
\]

The value of \(u\) must be restricted between 0 and 1, because duty-cycle is only defined between this range of values, and control law obtained from (13) is bounded using a saturation function.

**Observer-Based Controller (OBC)**

For OBC, desired system is proposed as

\[
\begin{align*}
\dot{x}_1 &= \frac{R_s}{L_s} x_1 - \frac{\text{sign}(v_s)}{L_s} (1-u) (x_2 + x_1) + \frac{v_s}{L_s} + \frac{k_1}{L_s} e_{s1} + \frac{k_2}{L_s} e_{s2} + \frac{k_3}{L_s} e_{s3} \\
\dot{x}_2 &= \text{sign}(v_s) (1-u) \frac{x_1}{C_1} - \frac{x_2}{R_1 C_1} \\
\dot{x}_3 &= \text{sign}(v_s) (1-u) \frac{x_1}{C_2} - \frac{x_3}{R_2 C_2} \ldots(14)
\end{align*}
\]

with \(e_{s1} = \hat{x}_1 - x_1^*, e_{s2} = \hat{x}_2 - x_2^*\) and \(e_{s3} = \hat{x}_3 - x_3^*\) are tracking errors using estimated variables. Tracking error dynamics are given as
To prove stability of observer-based controller, Lyapunov function candidate is proposed as

$$V(e, e_r) = \frac{1}{2} \left[ L_s (e_1^2 + e_2^2) + C_1 (e_2^2 + e_3^2) + C_2 (e_3^2 + e_3^2) \right],$$

whose derivative along trajectories of (15) is

$$\dot{V}(e, e_r) = L_s (e_1 \dot{e}_1 + e_2 \dot{e}_2) + C_1 (e_2 \dot{e}_2 + e_3 \dot{e}_3) + C_2 (e_3 \dot{e}_3 + e_3 \dot{e}_3)$$

$$\dot{V}(e, e_r) = -R_s e_1 \dot{e}_1 - \frac{\text{sign}(v_s)}{L_s} (1 - u)(e_2 + e_3) - \frac{R_s}{L_s} e_1$$

$$\dot{e}_1 = -\frac{R_s}{L_s} e_1 - \frac{\text{sign}(v_s)}{L_s} (1 - u)(e_2 + e_3) - \frac{R_s}{L_s} e_1$$

$$\dot{e}_2 = \text{sign}(v_s)(1 - u) \frac{e_1}{C_2} - \frac{e_2}{R_s C_2}$$

$$\dot{e}_3 = \text{sign}(v_s)(1 - u) \frac{e_1}{C_2} - \frac{e_3}{R_s C_2}$$

$$\dot{e}_1 = \frac{R_s}{L_s} e_1 - \frac{\text{sign}(v_s)}{L_s} (1 - u)(e_2 + e_3) - \frac{R_s}{L_s} e_1$$

$$\dot{e}_2 = \text{sign}(v_s)(1 - u) \frac{e_1}{C_2} - \frac{e_2}{R_s C_2}$$

$$\dot{e}_3 = \text{sign}(v_s)(1 - u) \frac{e_1}{C_2} - \frac{e_3}{R_s C_2}$$

...(15)

By means of algebraic manipulation, the derivative is expressed as

$$\dot{V}(e, e_r) = \left[ (R_s - \alpha_1) e_1 + \frac{1}{R} \right] e_1 + \left[ (R_s - \alpha_2) e_2 + \frac{1}{R} \right] e_2 + \left[ (R_s - \alpha_3) e_3 + \frac{1}{R} \right] e_3$$

$$\dot{V}(e, e_r) = -\alpha_1 e_1^2 - \alpha_2 e_2^2 - \alpha_3 e_3^2$$

... (16)

with $0 < \alpha_1 < R_s$, $0 < \alpha_2 < \frac{1}{R}$ and $0 < \alpha_3 < \frac{1}{R}$ and $0 < \alpha_3 < \frac{1}{R}$.

Eq. (21) implies that proposed OBC (Fig. 2) is asymptotically and globally stable.
Simulation Results

Simulation results in Matlab 7.10 (R2010a) and Simulink 7.5 using solver ode14 with fixed-step and step size of 5 X 10-5 for a 1kVA single-phase AFE rectifier are presented to evaluate close-loop performance of OBC. Active rectifier parameters are given as: power, 1 kVA; line voltage, 127 V_{RMS}; DC voltage, 200 V regulated; DC capacitors ($C_1$ & $C_2$), 2200 μF; inductor ($L_0$), 5.0 mH; and associated inductor resistance ($R_s$), 1.0 Ω. Controller and observer gains are established under constrains (16) and are given in Table 1. Under operating conditions of simulated AFE rectifier (Fig. 3a), AC main voltage $v_\text{s}$ suffers a 45% voltage sag between 1.0 s and 2.0 s. Load current $i_{\text{load}}$ is subject to a load step (Fig. 4b). In Luenberger-like observer performance test, measured (actual) and estimated currents are shown (Fig. 4a), besides maximum estimation error (3.8%, Fig. 4b). Measured and estimated currents (Fig. 5) are almost sinusoidal waves [total harmonic distortion (THD),

\begin{table}[h]
\centering
\begin{tabular}{ |c|c|c| } 
\hline
\textbf{System} & \textbf{Parameter} & \textbf{Value} \\
\hline
 Observer & $[k_{11} \ k_{12}]$ & $[0 \ 0]$ \\
 & $[k_{21} \ k_{22}]$ & $[10 \ 0]$ \\
 & $[k_{31} \ k_{32}]$ & $[0 \ 10]$ \\
 Control & $R_{P1}$ & 200 \\
\hline
\end{tabular}
\caption{Observer and control gains}
\end{table}

Fig. 3—Operating conditions of AFE rectifier: a) AC voltage under sag; and b) Nonlinear load current with load step

Fig. 4—Observer performance test: a) Measured (actual) and estimated currents; and b) Maximum estimation error.

Fig. 5—Sinusoidal waves with total harmonic distortion (THD).
12.5\%], indicating that PF is close to one. Thus, in spite of voltage sag and load step change, estimated current $\hat{x}_i$ perfectly follows the measured current $x_i$. Looking into performance of controller (Fig. 6a), voltage $V_{BUS}$ is
regulated at 200V and PF (mean value, 97.5%) is unity (Fig. 6b). In spite of using an observer-based scheme, the performance is similar to reported one\textsuperscript{4}, where it is necessary to measure all state variables.

**Conclusions**

AFE rectifiers embedded in industry applications are required to achieve simultaneously multiple tasks (power factor correction, harmonic cancellation, voltage sag ride through, etc.) with good performance. In this study, a closed-loop stability analysis using an observer-based passivity controller was applied to a single phase AFE rectifier; proposed observer is a Luenberger-like one due to its simplicity. Proposed scheme has capability to work without certain measurements, then it can operate safely even with sensors failures. Simulation results are presented for a 1kVA AFE rectifier with dynamical load and voltage sag conditions.
References