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Analysis of space charge field of sheet electron beam for compact high power and high frequency microwave devices

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Importance of sheet beam technology and role of space charge field analysis for success of this technology have been studied. Space charge field of sheet electron beam has been investigated in three approaches. At first a simplified infinite width sheet and line model are given to investigate nature of variation of field, then analysis is done for finite width beam. Using a theoretical model in OPERA 3-D, simulation has been done to compare space charge field. Space charge field of sheet electron beam is compared with that for equivalent round beam. Interpretation is made about nature of space charge field and its impact to sheet beam transport. Importance of proper optimization of beam height has been emphasized.

Keywords: High Pf^2 microwave sources, Sheet beam technology, Space charge field analysis, Beam transport, Effect of tunnel height

1 Introduction

There is an emerging need of very high pf^2 microwave sources for the world, where p is power and f is frequency of output RF signal. Several MW to GW of microwave power in mm range is required for LINAC, controlled thermonuclear reaction, material technology and plasma chemistry reactors. Again several watts to kW of RF power in THz range is required for biomedical imaging, high data rate radar communication, astronomy, material technology and other next generation applications[1,2]. Round beam devices are unable to fulfil these demands because requirement of beam diameter less than equal to one-tenth of wavelength of RF signal limits the amount of transmitted beam current[1,3] and at very high voltage there is possibility of breakdown[4]. Sheet electron beam technology is the best solution to both of these problems. By keeping thickness constant and increasing width, very large amount of beam current can be transmitted, hence comparable low voltage is required which reduces the possibility of high voltage breakdown and size of tube. Hence sheet beam technology is comparable with vacuum micro-electronics and can be used to generate and amplify microwave signal with high pf^2 output. However development of sheet beam devices is arrested due to difficulty in beam transport up to long distance which arises due to potential beam instabilities, edge effect, non-uniformities and resistive wall instability[2,3,5-8]. To investigate proper focusing method, proper space charge field analysis is required. There is no detailed analysis of space charge field in available literature. In present paper a detailed analysis of space charge field is presented which is very useful to investigate proper focusing technique.

2 Space Charge Field of Sheet Electron Beam of Infinite Width

Space charge field (SCF) is the self electric field generated by electrons present in the beam which diverge the beam. As the surface of rectangular sheet beam is not Gaussian, it is very difficult to exactly determine the SCF. Hence to investigate behaviour of field, a simplified approximate model is considered. The beam cross section is shown in Fig. 1. In the analysis beam width, thickness and direction of beam transport are in x, y, z axis, respectively. To determine y-component of electric field the beam is assumed to

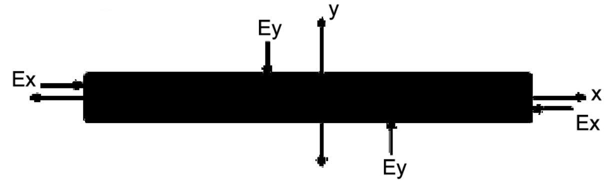

Fig. 1 — Cross-sectional view of sheet beam showing direction of electric field acting on it

be of infinite width and small thickness. Again to investigate variation of x-component of electric field a discrete linear charge distribution along its width is considered.

In this case the beam is assumed as infinite sheet of charge with available surface charge density (σ) at distance y from origin 'ρy' or '$\rho t/2$' depending upon internal or external position of investigation point. By using Gauss law of electrostatic, y-component of SCF can be expressed as:

$$Ey = -\sigma/\epsilon_0 = -Jy/u\epsilon_0 \text{ for } -t/2 < y < t/2$$
$$= -Jt/2u\epsilon_0 \text{ for } y > t/2 \text{ or } y < -t/2 \text{ or } y = \pm t/2 \quad \ldots(1)$$

Here J and u are current density and beam velocity, respectively. Above expression indicates that Ey varies linearly with y up to top and bottom of beam then remains constant outside of beam. To investigate variation of x-component of field, it is assumed that field is independent of z and charge is distributed in discrete manner along beam width. Assumption of the discrete linear distribution of charge along the beam width has justification for z-independent potential and very thin beam. Although this model does not give the exact field but is capable of investigating nature of variation of field. There are ($2n$+1) charges of effective magnitude Q_L distributed along beam width with distance of separation 'Δx' as shown in Fig. 2. Here beam width is considered as function of 'n' only. For the central charge i=0. The rest charges in both direction of x-axis have i=1 to n, respectively from the position of central charge.

Using coulomb's law and principle of super-position x-component of electric field at $x = p\Delta x$ can be expressed as:

$$Ex = [-Q_L/4\pi\epsilon_0 (\Delta x^2)]\, \Sigma\, (n+1-i)^{-2} + (n+i)^{-2}$$

$$\approx [-Q_L/4\pi\epsilon_0 (\Delta x^2)]\, \Sigma\, (n+1-i)^{-2} \quad \ldots(2)$$

Here summation is over i=0 to $i=p$.

The series in above expression tells that Ex at central region is almost zero and is significant near beam edge only. At beam edge the series reduces to $\Sigma 1/n^2$ which converges to $\pi^2/6$. Hence at beam edge

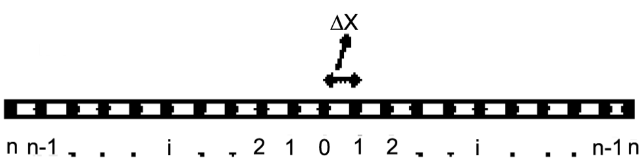

Fig. 2 — Discrete charge distribution along beam width

Ex is independent of n. Hence, SCF at beam edge is independent of beam width. Variation of Ex with x is shown in Fig. 3.

3 SCF for Sheet Electron Beam Transporting through Drift Tube Tunnel

Consider a finite width sheet beam of width w, thickness t, current density J, and volume charge density ρ, transmitting through a drift tube of tunnel height d and very large width a as shown in Fig. 4.

Now potential due to charge on beam is solution of Poison's equation given by:

$$\nabla^2\Phi = -\rho/\epsilon_0;\ \Phi(x=\pm d/2) = \Phi(x = \pm a/2) = 0 \quad \ldots(3)$$

Assuming the solution is independent of z and using Green's function solution method, and taking gradient of above potential, SCF in x and y-direction can be approximated as[2]:

$$Ex = (-2m\omega_p^2/\mathrm{de})\, \Sigma\{\sin(k_j t/2)\exp(-k_j w/2) \times \cos(k_j y)\sinh(k_j x)/k_j^2\} \quad \ldots(4)$$

$$Ey = (-4m\omega_p^2/de)\Sigma\{\sin(k_j t/2)\sin(k_j y)/k_j^2\} \times \{1-[\exp(-k_j(w/2+x)) + \exp(-k_j(w/2-x))]/2\} \quad \ldots(5)$$

Here $\omega_p = (e\rho/m\epsilon_0\gamma^3)^{1/2}$ is plasma frequency, e, m are respectively charge and mass of electron, $k_j = (2j+1)\pi/d$. The summation is over j=0 to infinite.

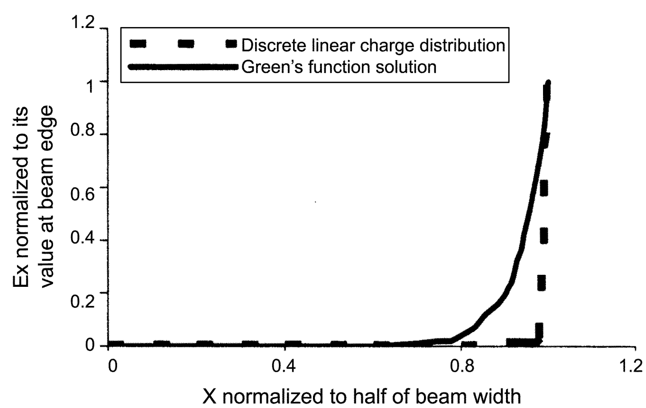

Fig. 3 — Variation of x-component of space charge field with x

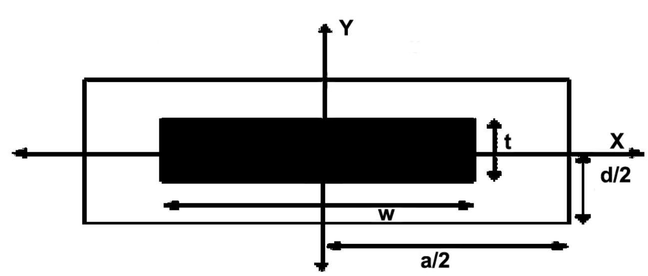

Fig. 4 — Sheet beam cross-section through drift tube tunnel

Image charge effect of side walls is neglected in above expressions and are valid if side walls are not very close to beam.

For infinite width beam, $w/2$ tends to infinite; except beam edge, Eq. (5) reduces to:

$$Ey=(-4m\omega_p^2/de)\Sigma\{\sin(k_j t/2)\sin(k_j y)/k_j^2\} \qquad ...(6)$$

Equations (4)-(6) are solved by MATLAB programming. Equation (4) converges for j=30 and Eqs (5) and (6) converges for j=1000 with relative convergence of order 10^{-5}. Variation of Ex and Ey with different position, beam thickness and width are shown in Figs 3, 5-9. In all figures except Fig. 9, normalized values are used. These curves are standard for any beam parameter of beam if beam width and drift tube width are comparable very large. Solution by MATLAB programming for beam of thickness 0.1 mm, current density 100 A/cm^2, beam kinetic energy 20 keV, tunnel height 1 mm confirms that Eqs (1) and (6) are identical which is shown in Fig. 9.

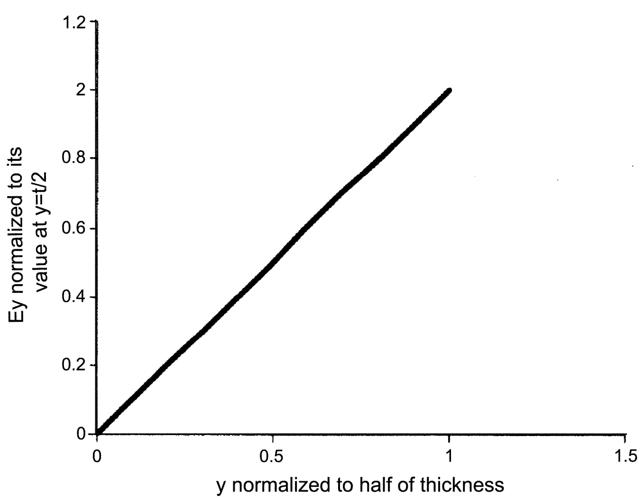

Fig. 5 — Variation of y-component of space charge field with y

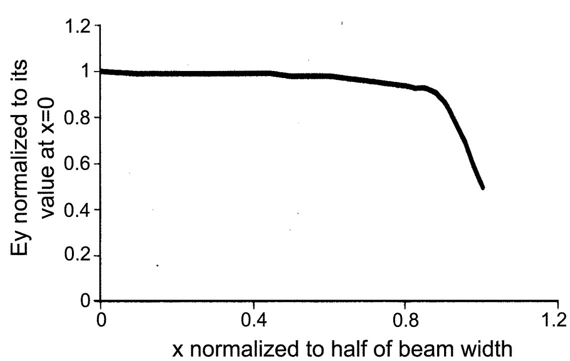

Fig. 6 — Variation of y-component of SCF with x

4 Simulation of Space Charge Field by OPERA 3D

OPERA 3D is a software which enables to model any complex structure and simulate electromagnetic problems[9]. Its SCALA solver solves Poison's equation by Finite Element Method (FEM). A

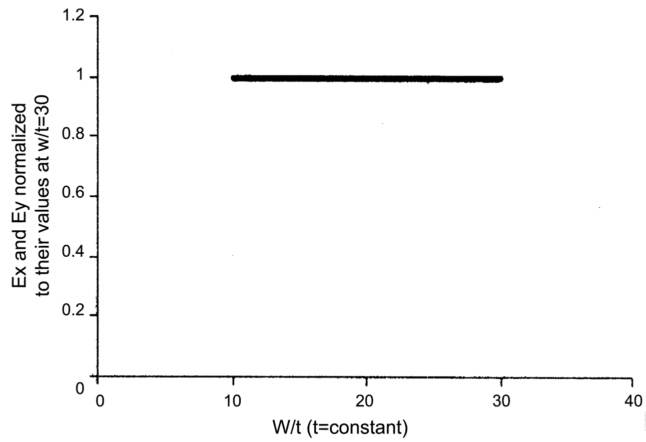

Fig. 7 — Variation of Ex, Ey with beam width

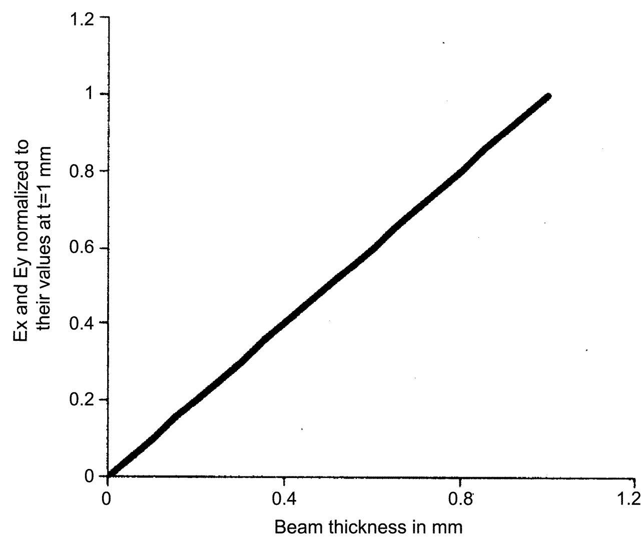

Fig. 8 — Variation of SCF with thickness

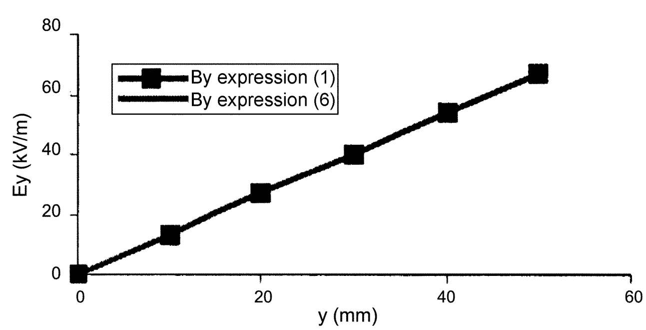

Fig. 9 — Comparison of variation of magnitude y-component of SCF for infinite width sheet beam

theoretical model is developed in this software by taking specified current density and kinetic energy emitter. Here the emitter behaves as the beam waist position of the actual electron gun. In this model, a sheet beam of kinetic energy 20 keV, current density 100 A/cm^2, width 20 mm, thickness 1 mm is transmitted through a drift tube of tunnel height 1.8 mm and width 28 mm. Mesh size of 0.1 mm is taken in all direction. Variations of x-component of SCF with x and y-component of SCF with y at z = 1 mm as per simulation using SCALA solver are shown in Figs 10-11. Comparison between simulated results and analytical results are shown in Figs 12-13. Again different models are developed in the software by varying tunnel height and width keeping other parameters constant. Variation of Ex with height fill factor (t/d) and width fill factor (w/a) is shown in Fig. 14. In all the figures magnitudes of SCF are considered.

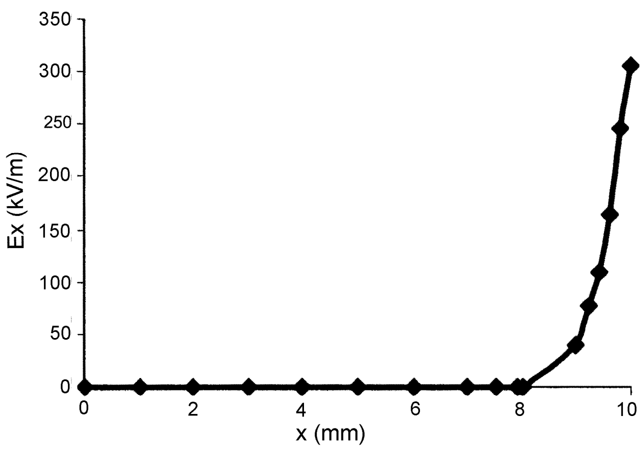

Fig. 10 — SCALA simulation result showing variation of magnitude of x-component of SCF with x

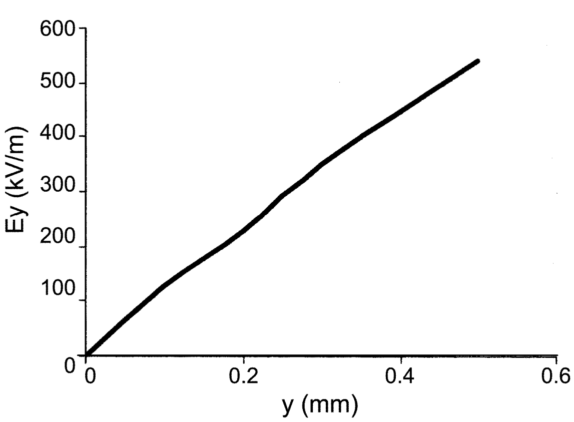

Fig. 11 — SCALA simulation showing variation of magnitude of y-component of SCF with y

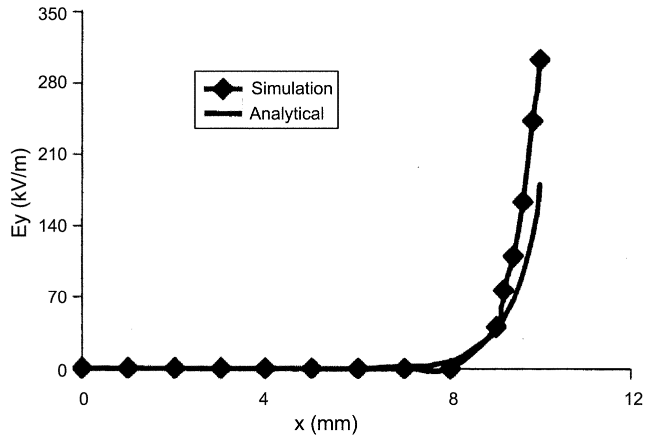

Fig. 12 — Comparison between simulation and analytical result for variation of x-component of SCF with x

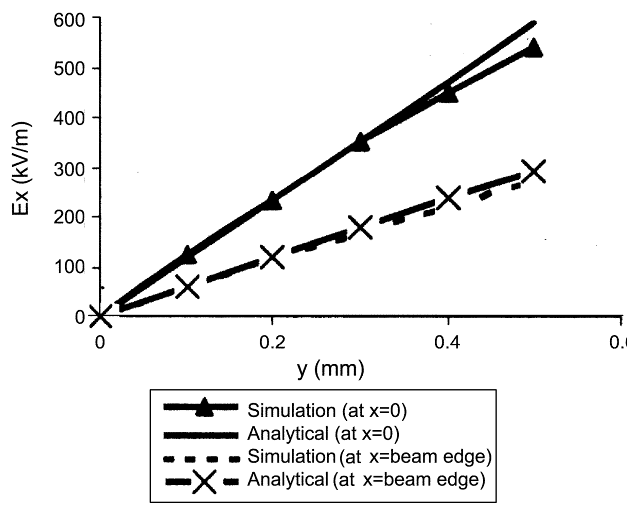

Fig. 13 — Comparison between simulation and analytical result for variation of Ey with y

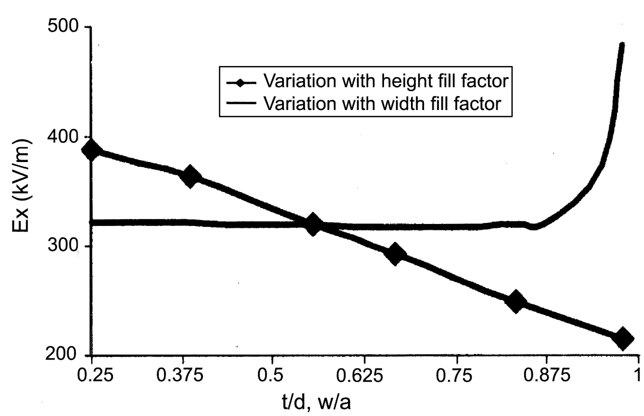

Fig. 14 — Variation of x-component of SCF with height fill factor (t/d) for w=28 mm, width fill factor (w/a) for d=1.2 mm

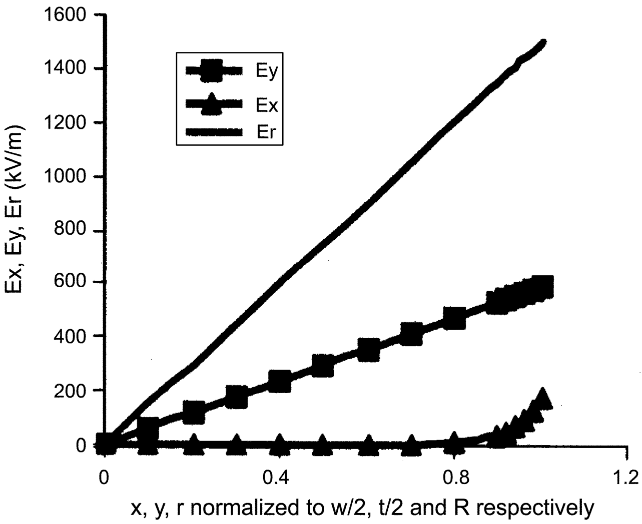

Fig. 15 — Comparison of SCF between sheet and round electron beam

5 Comparison of Space Charge Field of Round and Sheet Beam

For this purpose, a sheet beam of width 20 mm, thickness 1 mm, current density 100 A/cm^2, beam current 20 A, beam kinetic energy 20 keV is considered. This beam is converted into an equivalent round beam such that beam current, current density, beam kinetic energy remain same in both cases. The equivalent beam radius is $R= (20/\pi)^{1/2}$ mm. Radial space charge electric field for round beam at distance r from beam axis can be expressed as:

$$E_r = -\rho r/2\epsilon_0 = -Jr/2u\epsilon_0,\ r = R \qquad \ldots(7)$$

The comparison of SCF is shown in Fig. 15. The comparison result shows that space charge field for sheet beam is very much less than equivalent round beam. In this case $Ex<Ey<Er$.

6 Interpretation

By converting a round beam into equivalent sheet beam, SCF reduces significantly. SCF for sheet beam is independent of beam width but linearly varies with thickness. By increasing beam width and decreasing thickness SCF can be reduced significantly. As a result, magnetic field strength required to focus the beam reduces and the device will be more compact. However, several instabilities are noticed in sheet beam which generally not occur in round beam due to peculiar behaviour of SCF in sheet beam. *Ex* suddenly increases near beam edge for sheet beam which leads to edge effect. Due to $Ey\times B/B^2$ velocity drift between top and bottom layers, the beam is under shear and edges of beam curl. Both of these effects cause instability in sheet beam that is called diocotron instability. Reduction of *Ey* near beam edge leads to emittance growth.

Ex reduces by increasing in height fill factor (t/d). Hence tunnel height is the key parameter for stable beam transport. If the height is kept very less, then non-uniformity of electric field will increase and scalloping amplitude increases. Hence the optimized tunnel height is the crucial factor to minimize both diocotron instability and scalloping amplitude due to non-uniformity term.

7 Conclusions

Behaviour of space charge field of sheet electron beam and effect of tunnel size on it is investigated analytically and numerically. Space charge field of sheet electron beam has different behaviour along narrow and wide dimensions. Component of SCF along narrow dimension(y-component) of beam varies linearly with y up to beam surface. But component of SCF along wide dimension (here x-component) shows peculiar behaviour in its variation with x, unlike that for equivalent round beam. Magnitude of this component is almost zero except near beam edge and increases sharply near beam edge. Space charge field of sheet electron beam is decided by beam thickness only and not by the beam width. Hence by converting a round beam into equivalent wide sheet beam SCF can be reduced significantly. SCF is independent of tunnel width except the side walls of tunnel are very close to beam. When side walls of tunnel are kept very close to beam, component of SCF along wide dimension increases sharply which is due to image charge effect. By increasing height fill factor i.e. by decreasing tunnel height, x-component of SCF decreases. Above analysis is very helpful for investing proper focusing technique which is the future work of authors.

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