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Review Article

Quantum Hall effect in graphene: Status and prospects

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Graphene is the recently discovered two-dimensional (2D) one atom thick allotrope of carbon. Electrons in graphene, obeying a linear dispersion relation, behave like massless relativistic particles. It is a 2D nanomaterial with many peculiar properties. It is the thinnest material in the universe and the strongest ever measured. Its charge carriers exhibit intrinsic mobility and can travel micrometer-long distances without scattering at room temperature. Its unconventional Landau level spectrum of massless Dirac fermions leads to a new type of integer quantum Hall effect (IQHE) [known as half-integer quantum Hall effect or anomalous quantum Hall effect] which remains visible up to room temperature. Although, the conclusive evidence for collective behaviour of electrons in graphene is lacking so far, recently scientists have observed the fractional quantum Hall effect (FQHE) in graphene experimentally. In this review article, we discuss the IQHE and the FQHE in graphene briefly.

Keywords: Quantum Hall effect, Graphene, Landau levels

1 Introduction

Graphene is the recently discovered two-dimensional allotropic from of carbon. Before graphene, three-dimensional (diamond and graphite), one-dimensional (carbon nanotubes) and zero-dimensional (fullerenes) allotropes of carbon were known[1]. Two carbon allotropes-diamond and graphite– have been known to humans since ancient times. Fullerenes[2] were discovered in 1985, carbon nanotubes[3] in 1991 and graphene[4,5] in 2004. Since its discovery, graphene has become one of the hottest topics of research in materials science, physics and chemistry. It has unique electronic properties that researchers are eager to measure.

Graphene is a monolayer of carbon atoms packed into a dense honeycomb crystal structure. Graphene sheets are one-atom thick, 2D layers of sp^2-bonded carbon. In graphene, carbon atoms are arranged in hexagonal structure has two atoms per unit cell. The carbon-carbon bond length in graphene is approximately 1.42 Å, 1 Å (angstrom) = 0.1 nanometer or 1×10^{-10} meters. For each carbon atom on the lattice, three of four outer electrons get strongly bonded with its neighbouring atoms by σ orbitals[6]. The $2p_z$ orbital of the fourth electron produces a π bond with a neighbouring carbon atom. The σ bonds form the covalent structure with a honey geometry. These bonds strength provide the flexibility and robustness for the lattice geometry. On the other hand, the π bonds describe the intrinsic electronic structure of graphene. The π bands are decoupled from the σ bands because of inversion symmetry and closer to the Fermi energy. The Fermi energy separates occupied and empty states. In a neutral graphene sheet, this is equal to zero energy since valence and conduction bands meet (known as neutrality point). The bands form conical valleys that touch at two points (denoted by their momentum vector K and K') in the Brillouin zone, which makes graphene a gapless semiconductor. These points are named as Dirac points. Each π bond yields the half-filled electrons of p orbital to tunnel from a carbon atom to the neighbouring one. Thus, graphene is regarded as a many body system on which electrons can get correlated from site to site, resulting in a rich collective behaviour. The correlated behaviour can be represented by quantum effects which can influence on graphene's electronic properties[7].

Graphene is considered as the mother of all graphitic materials[8,9] because it is the building block for carbon materials of all other dimensions (Fig. 1). Graphite is obtained by the stacking of graphene layers. Diamond can be obtained from graphene under extreme pressure and temperatures by transforming the 2-dimensional sp^2 bonds into 3-dimensional sp^3 bonds. Graphene can be wrapped into zero-dimensional fullerenes and can also be rolled into one-dimensional carbon nanotubes.

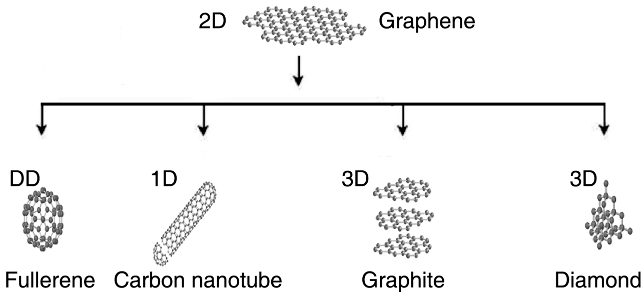

Fig. 1 — Formation of 0D, 1D and 3D carbon materials from graphene

Graphene is the first 2D material which is considered as the most attractive 2D nanomaterial[10,11] because of its superior material properties. It is the thinnest material in the universe and the strongest ever measured in terms of Young's modulus and elastic stiffness (the only other material that is comparable in strength is diamond). Its charge carriers exhibit intrinsic mobility, have the smallest effective mass (it is zero) and can travel micrometer-long distances without scattering at room temperature[12]. It has excellent electrical, thermal and optical properties. Electrons in graphene obey a linear dispersion relation i.e. $E = \hbar k v_F = p v_F$, where $p = \hbar k$ is momentum and v_F is the velocity of electrons in graphene, known as Fermi velocity, $v_F \sim c/300 = 10^6 m s^{-1}$. Here, $E \alpha p = \sqrt{p^2 + 0}$ implies the effective rest mass is zero [Since, the energy of a particle having rest mass m_0 and moving with a velocity u in a medium is $E = \sqrt{p^2 u^2 + m_0^2 u^4}$]. Thus, the charge carriers in graphene have zero effective mass and move at a constant velocity. Electrons in graphene are not actually massless. The effective mass is a parameter that describes how an electron at particular wave vectors responds to applied forces. Since the velocity of electrons confined on graphene remain constant that indicates that the parameter (effective mass) vanishes.

Whether graphene is a semiconductor or a metal? There are different views regarding this matter (a) Graphene has often been called a zero-gap semiconductor because the density of states is given by $D(E) = |E| / 2\pi\hbar^2 v_F^2$, which vanishes at $E = 0$. But it is observed that the conductivity of graphene is independent of the Fermi energy and the electron concentration as long as variations in effective scattering strength are neglected. Hence, graphene is regarded as a metal[13] rather than a zero-gap semiconductor. (b) Usually metals require only one energy band to describe them but semiconductors require two energy bands (conduction band and valance band) and an energy gap between them. Graphene has two bands, one for particles which is empty and other for antiparticles (holes) which is filled, but there is no gap between the two bands. That is why, graphene is considered as a hybrid[14] between a metal and a semiconductor. (c) According to some scientists, graphene is a semi-metal[15].

2 Integer Quantum Hall Effect in Graphene

Dirac's theory explains why electrons have spin. But when it is applied to graphene, the theory shows the electrons have pseudospin[16] due to peculiar symmeteries of the honeycomb lattice. This pseudospin protects graphene electrons against backscattering off defects and impurities. This enhances the conductivity and explains why charge carriers in graphene tend not to localize.

Graphene shows very interesting behaviour in the presence of a strong perpendicular magnetic field at low temperatures. In the presence of a perpendicular magnetic field B, electrons (holes) confined in two dimensions are constrained to move in close cyclotron orbits that in quantum mechanics are quantized. The quantization of cyclotron orbits is reflected in the quantization of the energy levels: at finite B, the $B = 0$ dispersion is replaced by a discrete set of energy levels, known as Landau levels (LLs). In other words, we can say that electrons occupy discrete Landau energy levels as a result of their quantized orbits. That quantum behaviour shows up as plateau in the conductance measured transverse to the current flow. As one Landau level fills up, the conductance is flat, showing no increase with carrier density until the next Landau level is nearly filled. The plateau appear at conductance values[17] $\sigma_{xy}=\nu e^2/h$, where ν is a Landau level filling factor which takes on integer values in the case of IQHE. The filling factor is defined as the ratio of the number of charges to the number of magnetic flux lines. In case of graphene, its unconventional Landau level spectrum of massless chiral Dirac fermions leads to a new type of integer quantum Hall effect[18,19] and the Hall conductivity $\sigma_{xy}=\pm 4e^2/h(n+1/2)$ in a magnetic field of 10 T and a temperature 1.6 K, where n is the Landau level index and the factor 4 accounts for graphene's double spin and double valley degeneracy. The shift of ½ originates[20,21] from the Berry phase due to the pseudospin (or valley) precession when a massless Dirac particle executes cyclotron motion. This unconventional QHE forms a series of filling factors $\nu=\pm2, \pm6, \pm10,\ldots$ [Whereas for non-relativistic two-dimensional electron systems (2DES) or standard 2DES, the Hall conductivity, $\sigma_{xy}=\pm4ne^2/h$]. That is why; it is characterized as half-integer quantum Hall effect or anomalous quantum Hall effect. The additional ½ is the hallmark of the chiral nature of the Dirac fermions in graphene. The first plateau occurs at $2e^2/h$ [$=(1/2)(4e^2/h)$] which is absent in non-relativistic 2DES. This is a special status of $n = 0$ Landau level for the massless fermions: half of its states are hole states and the other half are electron states. This anomalous QHE is the direct evidence for Dirac fermions in graphene.

The QHE in bilayer graphene is more interesting. Bilayer graphene consists of two weakly, Van der Waals coupled honeycomb sheets of covalent bond carbon atoms. In bilayer graphene, the Landau level spectrum is composed of eight-fold degenerate states at the zero energy and fourfold ones at finite energies under the high magnetic field. This can allow us to observe the quantum Hall plateaus at a series of $\nu=\pm4, \pm8, \pm12,\ldots\ldots$ In this case, the first plateau at $n = 0$ is absent and the first plateau appears at $4e^2/h$ just like the conventional QHE in semiconductor based 2DES. In general, the quantized plateaus appear at the standard sequence $\sigma_{xy}=\pm4e^2/h$ (same as the non-relativistic electrons) with a missing plateau at zero energy[19], so that the step in the Hall conductance separating electron- and hole-like regions is twice as large as the quantized steps on either side of the charge neutral state. This anomaly at $E = 0$ can be removed by field effect doping, which has the effect of adding carriers and splitting the layer degeneracy of the zero energy Landau level producing two new steps in the Hall conductance, each of height $4e^2/h$. This unusual quantization in bilayer graphene leads new elementary excitations called massive Dirac fermions. These fermions have quadratic dispersion like massive non-relativistic particles[22,9]. Thus, the anomalous IQHE in single layer graphene is associated with massless Dirac fermions and a zero energy mode (leading to a zero energy Landau level in the presence of a magnetic field) whereas the anomalous IQHE in bilayer graphene is associated with massive Dirac fermions and two zero energy modes. Further, it is also found that room-temperature [23]QHE can be observed in graphene. The observation of QHE in graphene at room temperature opens up new perspectives for graphene based quantum devices.

Very recently, the authors[24] have studied the quantum anomalous Hall effect in single and gated bilayer graphene systems in the presence of both exchange fields and spin-orbit coupling (Rasha spin-orbit coupling). They find that the quantized anomalous Hall conductivity in bilayer graphene is twice that of single layer graphene when the gate voltage is smaller than the exchange field, otherwise it is zero.

3 Fractional Quantum Hall Effect in Graphene

Graphene is now exciting scientists with its unusual material properties. Theoretical scientists expect that electrons in graphene are strongly interacting and hence exhibit fractional quantum Hall effect (FQHE). But surprisingly, the evidence for collective behaviour of electrons in graphene is lacking so far. The IQHE can be understood solely in terms of individual electrons in a magnetic field whereas the FQHE can be understood by studying the collective behaviour of all the electrons. The FQHE requires lower temperature, higher magnetic field and higher mobility compared with the IQHE. The IQHE do not depend upon interactions between electrons whereas the FQHE depends upon the combined effects of the magnetic field and Coulomb interaction between electrons. The FQHE has not yet been conclusively observed experimentally in graphene but it has been explored theoretically in a number of papers[25–29]. The possible fractional quantum Hall states in graphene have been discussed in those papers. Recently, we[30,31] have studied the FQHE in graphene theoretically. The FQHE reflects new physics arising from the collective behaviour of all the electrons and the FQHE of electrons can be considered as an IQHE of the composite particles. The quantized Hall conductivity (σ_{xy}) in FQHE in graphene is found to be:

$$\sigma_{xy} = \pm \frac{2p+1}{2m(2p+1)+1}\frac{2e^2}{h}$$

where $m, p = 0, 1, 2 \ldots$

The variation of σ_{xy} with respect to different values of filling factors (ν) is shown in Fig. 2, where filling factor (ν) is defined as $\sigma_{xy}\equiv\nu(e^2/h)$. The unusual electronic dispersion of graphene is reflected in the variation of Hall conductance staircase. The localization of electrons and quasi particles is believed to be responsible for the formation of the plateaus in the Hall conductivity. Due to the special lattice structure of graphene and Dirac nature of

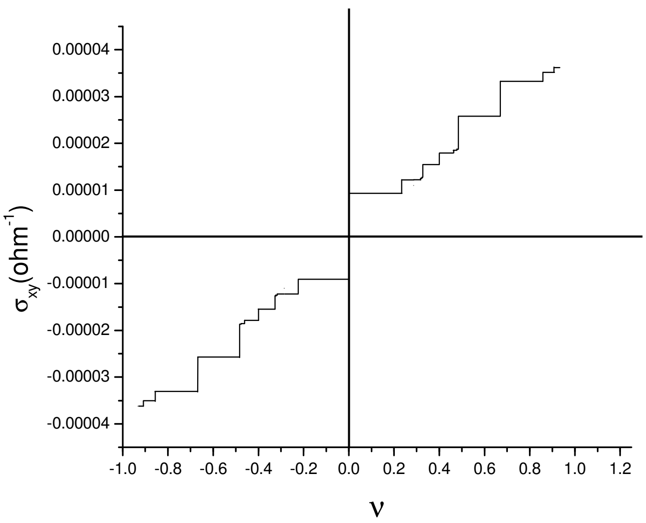

Fig. 2 — Quantized Hall conductivity (σ_{xy}) of graphene in fractional quantum Hall effect for different filling factors (v)

carriers, the edge states play a key role in quantum Hall transport. Our theoretical study makes new predictions that may be checked experimentally in future. The observation of FQHE in graphene would be the evidence that electrons are forming a collective state that has a fractional electric charge.

Recently, Du *et al.*[32] have showed that Dirac electrons exhibit strong collective behaviour which give rise to FQHE in graphene. They had suspended the graphene sample between two supports. Electrons in suspended samples have higher mobilities than those in samples resting on a substrate. For example, at low carrier density, the mobility in suspended graphene can exceed 2,00,000 $cm^2V^{-1}s^{-1}$, more than 10 times greater than those seen in non-suspended samples. They measured the Hall effect with two-probe method to get the better results than the conventional four-probe technique. They observed plateau in the transverse conductance at 1/3 of the conductance quantum e^2/h. They find that the FQHE can be observed at low temperatures in the magnetic fields as low as 2 T and can persist up to 20 K in a field of 12 T. This is more clear than in the semiconductor based 2DES which indicates the stronger Coulomb interaction and the more 2D nature of the 2DES in graphene.

Recently, Bolotin *et al.*[33] have also used similar techniques to find fractional 1/3 quantum Hall state in ultra clean suspended graphene, along with hints of plateaus at other fractional values of the conductance quantum. They used magnetic fields B up to 14 T and temperatures between 2 and 15 K. At B = 14 T, they observed the fractional quantum Hall state at v=1/3 plateau at electron density $n_c \approx 10^{11} cm^{-2}$. They have also extracted the quantum Hall states at v=0.46±0.02 and v=0.68±0.05 or equivalently v=1/2 and v=2/3 from their observations. It is expected that the next state v=2/5 will be observed with better quality specimens. Further, they showed that at low carrier density graphene becomes an insulator with a magnetic-field-tunable energy gap. The device is fully insulating $R(>10G\Omega)$ at magnetic fields $B > 5$ T and at filling factors $v<0.15$. Again at the highest magnetic field (B = 14 T) and low temperatures ($T <$ 10 K), the device is fully insulating, but at higher temperatures $R(n=0.17\times10^{11}\ cm^{-2})\alpha \exp(E_A/2kT)$, where the activation energy $E_A \approx 60$ KK.

Very recently, the FQHE has been observed in a single-layer graphene sample fabricated on a hexagonal boron nitride (h-BN) substrate[34]. Hexagonal boron nitride is an insulating isomorph of graphite with boron and nitrogen atoms occupying the in equivalent A and B sublattices in the Bernal structure. It has an atomically smooth surface that is relatively free of dangling bonds and charge traps. It also has a lattice constant similar to that of graphite, and has large optical phonon modes and a large bandgap (5.97 eV). They have observed the FQHE at fractional filling factors v=1/3, 2/3 and 4/3 in the n = 0 Landau level and at v=7/3, 8/3, 10/3, 11/3 and 13/3 in the n = 1 Landau level. The observed fractions are in good agreement with the expected SU(4) symmetry of the single-particle level being broken spontaneously at all fillings at which the FQHE is observed and confirms the large strength of electronic interactions in graphene.

4 Conclusions

Graphene is a novel two-dimensional electronic system with remarkable features providing for a solid state realization of quantum electrodynamics, massless Dirac particles and an unconventional quantum Hall effect. Electrons in graphene behave as if they lose rest mass or neutrinos acquire electric charges[19,35]. The energy-momentum relation is linear like a photon or a phonon, but coefficient is Fermi velocity instead of c or c_S, where c is the velocity of light and c_S is the velocity of phonon which is equal to velocity of sound. The electronic structure can be described by 2D massless relativistic fermions in graphene. These massless fermions enable us to study topological effects on electronic properties of graphene.

The spectrum of massless Dirac fermions leads to anomalous integer quantum Hall effect in graphene with the existence of n = 0 quantized Landau level[36] shared equally between electrons and holes. Since n = 0 QHE is unique to graphene, the issue requires further experimental and theoretical investigation.

Although, the conclusive evidence for collective behaviour of electrons in graphene is lacking so far, recently two groups[32,33] of scientists have observed the 1/3 FQHE in suspended graphene experimentally. Very recently, the FQHE has been observed in a single-layer graphene sample fabricated on a hexagonal boron nitride substrate[34]. We hope these results will open the new routes for studying the collective behaviour of electrons and FQHE in graphene. Still, it is the wealth of new physics-observed and expected. Although we now have a reasonable theoretical and experimental understanding of QHE (both IQHE and FQHE) in monolayer graphene, much work should be done in bilayer and multilayer graphene. The study of quantum Hall effect in graphene is a very challenging and fascinating topic both theoretically and experimentally in condensed matter physics as well as quantum field theory.

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