Effect of structure parameters on transmission characteristics in one-dimensional photonic crystal

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According to the electromagnetic theory of light in one-dimensional photonic crystal and method of forbidden band edge, the characteristics of forbidden band for S-polarization light have been studied. For S-polarization state, the reflectivity for different wavelength and different incidence angles has been analyzed via simulation under optics-film theory. The relations of wavelength parameter (g), the refractive index and the reflectivity of the medium layer have also been analyzed. By using the Bloch theory, their relations are presented. The results show that the reflectivity and the forbidden band gap increase with the increase of the incidence angle and refractive index (n1) of one-dimensional photonic crystal which has a high value. These quality results will be of great helpful for the design of one-dimensional photonic crystal.

Keywords: Photonic crystal, Bloch theory, Resonator laser

1 Introduction

One-dimensional photonic crystal is made up of various medium with different refractive indexes in one-dimension periodically and the medium in each layer is uniform and isotropic. Meanwhile, the refractive index varies periodically in the vertical direction. The structure of the one-dimensional photonic crystal (Fig. 1) is similar to multilayer media film in optics. There is a forbidden band in one-dimensional photonic crystal, which is an important property of semiconductor. Many papers have been published for this characteristic of the photonic crystal 1-3. In this paper, by applying the characteristic matrix method and the band-gap edge analysis 4, we analyzed the TE-polarization of the light-wave (S-polarization light), which transmits in one-dimensional photonic crystal composed of two kinds of medium.

2 Transmission Characteristics of One-dimensional Photonic Crystal

By utilizing the optics-film theory 5-7, the light transmission characteristics in each layer of the medium can be expressed by the following characteristic matrix:

\[
\begin{bmatrix}
\cos \delta_j & \frac{i}{\eta_j} \sin \delta_j \\
\eta_j \sin \delta_j & \cos \delta_j
\end{bmatrix}
\]  

...(1)

where \( \delta_j = 2\pi n_j d_j \cos \theta_j / \lambda \), \( j = 1,2 \), \( \theta_j \) is the angle between the light-ray and the interface of the medium, \( \lambda \) is the wavelength of the incidence light in the air, \( n_j \) is the refractive index of one medium lay in one-dimensional photonic crystal, the refractive index in the air \( n_0 = 1 \). To TE-light (S-polarized light), \( \eta_j = n_j \cos \theta_j \), \( j = 1,2 \); To TM-light (P-polarized light), \( \eta_j = n_j / \cos \theta_j \), \( j = 1,2 \). Two medium layers form a basic cycle unit. The characteristic matrix is:

\[
M = \begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
= \begin{bmatrix}
\cos \delta_1 & \frac{i}{\eta_1} \sin \delta_1 \\
\eta_1 \sin \delta_1 & \cos \delta_1
\end{bmatrix}
\begin{bmatrix}
\cos \delta_2 & \frac{i}{\eta_2} \sin \delta_2 \\
\eta_2 \sin \delta_2 & \cos \delta_2
\end{bmatrix}
\]  

...(2)
where

\[ m_{11} = \cos \delta_1 \cos \delta_2 - \frac{\eta_2}{\eta_1} \sin \delta_1 \sin \delta_2 \]

\[ m_{12} = \frac{i}{\eta_2} \cos \delta_1 \sin \delta_2 + \frac{i}{\eta_1} \sin \delta_1 \cos \delta_2 \]

\[ m_{21} = \frac{i}{\eta_2} \sin \delta_1 \cos \delta_2 + i \eta_2 \cos \delta_1 \sin \delta_2 \]

\[ m_{22} = \cos \delta_1 \cos \delta_2 - \frac{\eta_2}{\eta_1} \sin \delta_1 \sin \delta_2 \]

If one-dimensional photonic crystal is constituted by \( N \) basic cycle units of this kind, the total characteristic matrix of this photonic crystal will be:

\[
\begin{pmatrix}
B \\
C
\end{pmatrix} =
\begin{pmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{pmatrix}^N \begin{pmatrix}
1 \\
\eta_0
\end{pmatrix}
\] ... (3)

\[
R = \left( \frac{\eta_0 B - C}{\eta_0 B + C} \right)^N \left( \frac{\eta_0 B - C}{\eta_0 B + C} \right) \] ... (4)

The research methods are similar to S-polarized light and P-polarized light in this paper, only S-polarized light has been studied. When the dielectric layers have eight basic cycle units (namely \( N = 8 \)), and the parameters meet the restriction of \( n_1 d_1 \cos \theta_i = \lambda_0 / 4 \), \( n_2 d_2 \cos \theta_i = \lambda_0 / 4 \) (\( n_1 = 2.38 \), \( n_2 = 1.38 \), the center wavelength of the incidence light \( \lambda_0 = 600 \text{nm} \)), the relation of reflectivity variation depending on the incidence angle \( \theta_0 \) and wavelength \( \lambda \) is shown in Fig. 2. The parameter \( g \) is the ratio of the wavelength in vacuum to that in the considered medium. A simple mathematical description is \( g = \lambda_0 / \lambda \). The variation of reflectivity depending on the incidence angle \( \theta_0 \) and \( g \) change is shown in Fig. 3. From Figs 2 and 3, the reflectivity \( R \) and its bandwidth increase when the incidence angle \( \theta_0 \) becomes larger.

Figure 2 shows the relation of the incidence angle, the transmission wavelength and the reflectivity of light energy. The reflectivity of the central wavelength is the largest for both sides of the central wavelength, the reflectivity becomes smaller gradually (Fig. 2). At the edge of the band-gap, the reflectivity tends to zero. When the wavelength shifts to the short wavelength part, the reflectivity oscillates periodically and enlarged as the incidence angle increases.

In Fig. 3, we can obtain the coincident results with the help of Fig. 2. When the parameter \( g \) was used as a standard, namely, the band-gap of the photonic crystal was measured by the frequency of light, the two band gaps are equal approximately. The variations of \( R \) with \( n_1 \) and \( g \) are shown in Fig. 4 for \( \theta_0 = 0 \), while all the other conditions are the same. It is novel that the reflection bandwidth is extended and the reflectivity becomes large as \( n_1 \) increases. Some small vibrations are at one side of reflection band-gaps on the curve of reflectivity.

Figure 4 shows the relation of the parameters \( g \) and the reflectivity. It is found that there is one band-gap on both sides of the center band-gap. In the vicinity of the center wavelength, the reflectivity is the largest. With the increase of the refractivity of the dielectric layers (\( n_1 \)), the reflectivity of the light energy in the central wavelength increases and there are also some reflection peaks of the energy which is Chopy in the non-band gap. Comparing it with Fig. 3, it is observed
that the effect of dielectric layers refractivity on its reflection coefficient is larger than on the incidence angle. Additionally, with increase of the dielectric layers refractivity \(n_1\), the effect of dielectric layers refractivity on the non-band gap becomes very small. If it applies on the Resonator of laser, arbitrary wide spectrum laser can be produced.

When one-dimensional photonic crystal is used in optical signal processing, the needed optical signal in the band-gap should be totally reflected, and the non-required optical signal be transmitted. With increase of one-dimensional photonic crystals periods, the reflection coefficient is enlarged, and the fabrication becomes more difficult. When the signal reflection coefficient of light remains of the same value, we can increase the incidence angle \(\theta_0\) and the dielectric layers refractivity \(n_1\) to reduce the periods of one-dimensional photonic crystal and the fabrication difficulty. When increasing the incidence angle, the energy peak of the reflection for the non-band gap is under small influence, as the vibration is smooth in Fig. 4. So in the engineering, the reflection coefficient of light signals can be enlarged by increasing the dielectric layers refractivity \(n_1\). In this way, the signal-to-noise ratio of the photonic crystal filter can be significantly optimized.

3 Characteristic of the Band gap using Bloch Theory

The one-dimensional photonic crystal band-gap under arbitrary incidence angle with Bloch theory has been analyzed. It is much simpler than the transmission matrix in calculating the width and the position of the band-gap.

From the electromagnetic theory in optics, we know that light-wave can be expressed by the wave function of vector-\(Z\).

\[
\Psi(z) = \begin{bmatrix} E(Z) \\ H(Z) \end{bmatrix}
\]

when the light passes one basic cycle unit of this one-dimensional photonic crystal, the wave function \(\Psi(z + d)\) satisfies the following relation:

\[
\Psi(z + d) = M \Psi(z) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \Psi(z + d)
\]

Following Bloch theorem in the infinite cycle system, we have:

\[
\Psi(z + d) = e^{-ikd} \Psi(z)
\]

where \(K\) is the vector of the Bloch wave. Combining Eqs (5), (6) and (7), we will have the following function:

\[
\cos(Kd) = \frac{m_{11} + m_{22}}{2} = \cos\delta_1 \cos\delta_2 - \frac{1}{2} \left( \frac{\eta_2}{\eta_1} + \frac{\eta_1}{\eta_2} \right) \sin\delta_1 \sin\delta_2
\]

when \(\left|m_{11} + m_{22}\right|/2 < 1\) (K is a real number), Bloch wave is a non-decaying wave. The band of one-dimensional photonic crystal corresponds to the conduction-band for the light. When \(\left|m_{11} + m_{22}\right|/2 > 1\) (K is plural), Bloch wave is a decaying wave. It corresponds to the band-gap for the light. When \(\left|m_{11} + m_{22}\right|/2 = 1\), it is at the edge of the band-gap. All these give us a clear explanation for the physical essence of the band-gap appearance.

By using Eq. (9), we can plot the curve of \(F\) versus \(\lambda_0\) (\(\lambda_0\) the wavelength of the incident light in vacuum), and the curve of \(F\) versus \(g\). Based on the above analysis, we know \(F > 0\) is the condition of band-gap in the curve. Meanwhile, we can easily confirm the position and the width of the band-gap from the curve.
are easily confirmed. From Eqs (2)-(4), we can calculate the reflectivity of the photonic crystal numerically. Fig. 5 and 6 show the variation of \( F \) versus \( \lambda_0 \) and the variation of \( F \) versus \( g \) while \( \theta_0 = 0^\circ, 45^\circ \) and \( 90^\circ \). Here, \( n_0 = 1 \), \( n_1 = 2.38 \), \( n_2 = 1.38 \), the center wavelength \( \lambda_0 = 600 \text{ nm} \) and other parameters satisfy the conditions of 
\[ n_1d_1 \cos \theta_1 = \lambda_0/4 \quad \text{and} \quad n_2d_2 \cos \theta_2 = \lambda_0/4. \]
Figure 7 shows that both the width and the height of the bandgap increase, when \( n_1 \) increases. Meanwhile, it is found that the effect of \( n_1 \) is much larger on the incidence angle \( \theta_0 \). Additionally, the characteristics of the band-gap figure and the reflectance figure have been compared. The higher is the band-gap, the larger is the reflectivity. Meanwhile, by comparing Figs (5-7), the width and the position of the band-gap for different incidence angles have been obtained. It can be easily understood that \( F \) is enlarged and the band gap becomes wider when incidence angle is increased. Fig. 8 shows variation of \( F \) versus \( g \) for different \( n_1 \), when \( \theta = 0^\circ \).

4 Conclusions

Based on optics-film theory, the relations of angle, wavelength and reflectivity have been studied. By simulating, their dependence on each other in three dimension figures has been plotted. It is found that the reflectivity \( R \) and the bandwidth of the reflection have been enlarged by increasing the incidence angle \( \theta_0 \). By the method of band-gap edge under Bloch theory, it was obtained that \( F \) increases and the band-gap becomes wider when the incidence angle increases.
This can reduce the numbers of layers of the basic cycle of one-dimensional photonic crystal. Meanwhile, we also found that the thickness of the dielectric film $d_j$ will change by increasing the incidence angle $\theta_0$ and the dielectric layers refractivity $n_i$ when the central wavelength of the reflectivity and bandwidth are in certain. We hope our work will be helpful in the fabrication of one-dimensional photonic crystals and the resonator of laser.

References