Analytical and numerical investigation of the effect of pulse shape of intense, few-cycles TM$_{01}$ laser on the acceleration of charged particles

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Received 28 June 2010; revised 4 February 2011; accepted 4 March 2011

The effect of the pulse-shape, carrier-envelope phase (CEP) and number of cycles of the ultra-intense, ultra-short few-cycles TM$_{01}$ laser propagating in vacuum on the acceleration of the electrons has been analytically and numerically investigated. The Gaussian, Lorentzian, and hyperbolic-secant temporal profiles of the laser pulse have been considered to get an insight into the scheme of acceleration of charged particles. A laser of intensity $\sim 10^{22}$ W/cm$^2$ is found to accelerate the on-axis electrons from rest to the GeV energy range. The numerical results show that a single cycle Lorentzian profile of the laser pulse may mainly be used to obtain the higher values of possible electron acceleration.

Keywords: Charged particle acceleration, Laser-driven accelerator, Carrier-envelope phase dependence, Ultra-fast few-cycle pulse propagation

1 Introduction

The ultrafast laser using chirped pulse amplification (CPA) technique has renewed interest in laser acceleration of charged particles$^{1,2}$. The CPA technique has resulted in ultrashort electromagnetic pulse, which may contain only a few optical cycles. It can be focused down to a single wavelength leading to the so-called $\lambda^3$-regime$^3$. Current laser intensities have reached up to $10^{22}$ W/cm$^2$ and more values are expected to be obtained in near future. Such intense lasers at the focus can provide electric and magnetic field intensities 100 GV/cm and 300 MG, respectively. This field gradient is much higher than that of the conventional accelerators which is less than 1 MV/cm. Thus laser acceleration of particles, especially electrons, has attracted significant research attention$^{4-10}$. However, the use of such extremely powerful laser beams in accelerating charged particles leads to the serious damage to the material of the guiding or wave slowing structure. In order to overcome this limitation one makes use of the intense axial fields associated with laser pulse propagation in free space. An estimate of this longitudinal component of the beam is obtained by integrating Gauss’s law along the propagation axis $z$, yielding:

$$E_z = -\int [\nabla \times E_{\tau}] \cdot dz = \frac{i}{k} \nabla \times E_{\tau} = -\frac{i}{k} E_{\tau}$$

where $w$ is the beam width. This equation gives an order of magnitude of the accelerating electric field in terms of $w$. The form of the equation indicates that the acceleration is greatest at the beam focus. In doing so, one encounters the fact that a plane wave propagating along the $z$-direction with the vector potential $A_z(x-ct)\hat{e}_z$ cannot accelerate the electron, a result in conformity with the Lawson-Woodward theorem$^{11}$. The net energy gain by the electron is zero. This result is also true for non-focused laser pulses. The electron can, however, gain net energy even in vacuum if the electron may obtain a larger energy near the focus in the accelerating phase and loose less energy at the deceleration phase. Thus an electron can gain net energy from the intense focused laser pulse in vacuum. This is the basic idea of the ponderomotive-force electron acceleration by an intense focused pulse laser$^{12,13}$. Initial experimental observations of the acceleration of free electrons in vacuum up to MeV energies by a high-intensity sub-picosecond laser pulse ($10^{19}$ W/cm$^2$, 300 fs) validate the basic idea of vacuum acceleration$^{14}$. The experimental data are in good agreement with relativistic motion of electrons in a spatially and temporally finite electromagnetic field, both in terms of maximum energy and scattering angle. Recently a new approach to electron acceleration is to make use of the axial field component of the very intense TM$_{01}$ Laguerre-Gauss laser beams, even though these beams have phase velocities exceeding the speed of light in vacuum$^5$. In experiments related to ultra-intense
lasers, controlling the pulse shape is important both in time and space domains. Recent theoretical studies discuss the importance of the laser pulse shape on the wakefield acceleration of electrons. These studies suggest that proper tailoring of the ultrashort laser pulse can lead to an optimum gain of electron energy.

In this paper, the effect of the temporal profile of the few-cycle ultra-short, ultra-intense transverse magnetic laser pulse on the acceleration of the electrons in free space has been studied. Temporal profiles of the laser pulses are chosen to be Gaussian, Lorentzian, and Hyperbolic secant. We adopt a general formulation which is applicable to paraxial pulses with different temporal profiles. Finally we apply this formulation to estimate the energy gain by the electrons lying on the axis as well as slightly displaced from the axis. The numerical results show that the energy gained by the electron in due course of acceleration depends on the field intensity, CEP, number of cycles, and pulse-shape.

2 Theory

2.1 Field properties of the TM01 laser pulse propagating in free space

The intensity profile of the TM01 mode laser is ring shaped. We consider the spatio-temporal envelope sustained by a plane wave carrier. The wave propagation of the electromagnetic field pulse is usually studied by making use of slowly varying envelope approximation (SVEA). On adopting the ansatz:

$$E = \text{Re} \left[ \left( \tilde{E}_z + \sum_{n=1}^{\infty} \tilde{E}_r^n \right) \exp \left( j \left( \omega_0 t - k_0 z \right) \right) \right]$$

(1)

where \( \omega_0 \) is of centre carrier frequency and \( k_0 \) is wave vector, one obtains longitudinal electric field as a function of transverse electric field:

$$\tilde{E}_z = \sum_{m=0}^{\infty} \left( - \frac{j}{k_0} \right)^{m+1} \frac{\partial^m}{\partial z^m} (\nabla \cdot \tilde{E}_r)$$

(2)

The total transverse electric field with carrier envelope phase \( \varphi_0 \) can be expressed as:

$$E_\perp = \text{Re} \left[ \tilde{E}_r^n + \sum_{n=1}^{\infty} \tilde{E}_r^n \exp \left( j \left( \omega_0 t - \varphi_0 \right) \right) \right]$$

(3)

where local coordinates \( \zeta \) and \( t' \) are related as:

$$t' = t - \frac{z}{c}, \quad \zeta = z.$$

The \( n \)th order transverse field can be expressed as:

$$\tilde{E}_r^n(r, \zeta, t') = \left( \frac{j}{\omega_0} \right)^n \frac{\partial^n}{\partial t'^n} \frac{\partial^{n+1}}{\partial \zeta'^{n+1}} \left( \frac{z'^n}{n!} \frac{\partial \tilde{E}_r^0}{\partial \zeta'} \right)$$

(4)

where \( n = 1, 2, 3, \ldots \).

In expression (4) the zeroth-order term is:

$$\tilde{E}_r^0(r, \zeta, t') = A \left( \frac{j z_r}{\bar{q}} \right)^2 \exp \left( -\frac{j k_0 r^2}{2 \bar{q}} \right) f(t')$$

(5)

\( A \) is normalizing constant, complex parameter \( \bar{q} = \zeta + j z_r \), Rayleigh distance \( z_r \) is given by

$$z_r = \frac{k_0 \rho_0^2}{2}.$$  \( f(t') \) is the temporal profile of the pulse envelope. For a pulse having \( p \)-cycles each of period \( T_o \), the pulse duration \( T \) can be expressed as \( T = p T_o \).

The first derivative of the complex amplitude \( \tilde{E}_r \) can be expressed as:

$$\frac{\partial \tilde{E}_r}{\partial z} = \frac{\partial \tilde{E}_r}{\partial \zeta} \frac{\partial \zeta}{\partial z} + \frac{\partial \tilde{E}_r}{\partial \zeta'} \frac{\partial \zeta'}{\partial z} + \frac{\partial \tilde{E}_r}{\partial \zeta'} \frac{\partial \zeta'}{\partial z} \frac{\partial \zeta'}{\partial \zeta'}$$

Using the variables \( t' = t - \frac{z}{c}, \quad \zeta' = z \),

and using paraxial approximation \( \frac{\partial \tilde{E}_r}{\partial \zeta'} = 0 \), we get:

$$\frac{\partial \tilde{E}_z}{\partial \zeta} = -\frac{1}{c} \frac{\partial \tilde{E}_r}{\partial \zeta'}$$

(6)

The \( n \)th order correction in longitudinal field can, therefore, be expressed as:

$$\tilde{E}_z^n = \left( \frac{-j}{k_0} \right)^m \sum_{m=0}^{\infty} \left( \frac{j}{\omega_0} \right)^m \frac{\partial^m}{\partial t'^m} (\nabla \cdot \tilde{E}_r^n)$$

(7)

\( n = 0, 1, 2, 3, \ldots \).

The \( m = 0 \) term is the contribution to the longitudinal electric field that originates from the carrier of the transverse electric field and \( m > 0 \) terms are contributions that come from the transverse field envelope. Thus the total longitudinal electric field can be written as:
\[ E_z = \text{Re} \left[ \tilde{E}_z^\omega + \sum_{n=1}^\infty \tilde{E}_z^{\omega_n} \exp \left( j (\omega_n t - \varphi_0) \right) \right] \]  

(8)

2.2 Effect of temporal profiles on acceleration of charged particles

For numerical estimation of the acceleration of the charged particles, the temporal pulse profiles \( f(t') \) considered are:

\[ f(t') = f_g(t') = \exp \left( -\frac{t'^2}{T^2} \right) \]  

(Gaussian),  

(9)

\[ f(t') = f_L(t') = \frac{1}{1 + \frac{t'^2}{T^2}} \]  

(Lorentzian),  

(10)

\[ f(t') = f_{L}(t') = \text{sech} \left( \frac{t'}{T} \right) \]  

(secant).  

(11)

Using Eqs (4)-(7) in Eq. (8), the longitudinal electric field correct up to second order can be expressed as:

For Gaussian profile

\[ E_z = \frac{A \exp \left( -m \cos \psi_s \right)}{k_0} \left[ G_1 + G_2 + G_3 \right] f_g(t') \]  

(12)

For Lorentzian profile

\[ E_z = \frac{A \exp \left( -m \cos \psi_s \right)}{k_0} \left[ L_1 + L_2 + L_3 \right] \]  

(13)

For Hyperbolic Secant profile

\[ E_z = \frac{A \exp \left( -m \cos \psi_s \right)}{k_0} \left[ S_1 + S_2 + S_3 \right] \]  

(14)

where

\[ G_1 = 2 \left( \frac{\rho_0}{\rho(z)} \right)^2 \left[ \frac{H_1 \left( \frac{t'}{T} \right)}{\omega_0 T^2} - 1 \right] Q_1 + \frac{H_1 \left( \frac{t'}{T} \right)}{\omega_0 T} Q_2, \]  

(15)

\[ G_2 = \frac{4 z}{z_R} \left( \frac{\rho_0}{\rho(z)} \right)^3 \left[ \frac{H_1 \left( \frac{t'}{T} \right) H_2 \left( \frac{t'}{T} \right)}{\omega_0 T^2} - \frac{H_3 \left( \frac{t'}{T} \right) H_4 \left( \frac{t'}{T} \right)}{\omega_0 T^3} \right] Q_1 - \frac{H_2 \left( \frac{t'}{T} \right) Q_2}{\omega_0 T^2} \]  

(16)

\[ G_3 = \frac{4 z}{z_R} \left( \frac{\rho_0}{\rho(z)} \right)^3 \left[ \frac{H_1 \left( \frac{t'}{T} \right) H_2 \left( \frac{t'}{T} \right) - H_3 \left( \frac{t'}{T} \right) H_4 \left( \frac{t'}{T} \right)}{\omega_0 T^2} \right] Q_1 - \frac{H_2 \left( \frac{t'}{T} \right) Q_2}{\omega_0 T^2} \]  

(17)

\[ G_4 = \frac{4 z}{z_R} \left( \frac{\rho_0}{\rho(z)} \right)^3 \left[ \frac{H_1 \left( \frac{t'}{T} \right) H_2 \left( \frac{t'}{T} \right) - H_3 \left( \frac{t'}{T} \right) H_4 \left( \frac{t'}{T} \right)}{\omega_0 T^2} \right] Q_1 - \frac{H_2 \left( \frac{t'}{T} \right) Q_2}{\omega_0 T^2} \]  

(18)

\[ L_1 = 2 \left( \frac{\rho_0}{\rho(z)} \right)^2 \left[ \frac{L_{12}}{\omega_0^2} - f_{L}(t') \right] Q_1 + \left( \frac{L_{13}}{\omega_0^3} - \frac{L_{11}}{\omega_0} \right) Q_2 \]  

(19)

\[ L_2 = \frac{4 z}{z_R} \left( \frac{\rho_0}{\rho(z)} \right)^3 \left[ \frac{2 L_{12}}{\omega_0^2} - \frac{L_{13}}{\omega_0} \right] Q_3 - \frac{L_{14}}{\omega_0^3} Q_4 \]  

(20)

\[ L_3 = \frac{24 \left( \frac{t'}{T} \right) \left( f_{L}(t') \right)}{T^6} - 48 \left( \frac{t'}{T} \right)^3 \left( f_{L}(t') \right)^4, \]  

(21)

\[ L_{11} = -2 \frac{t'}{T} \left( f_{L}(t') \right)^2, \]  

(22)

\[ L_{12} = \frac{8 \left( \frac{t'}{T} \right)^2 \left( f_{L}(t') \right)^3}{T^4} - 2 \left( f_{L}(t') \right)^2, \]  

(23)

\[ L_{13} = \frac{24 \left( \frac{t'}{T} \right) \left( f_{L}(t') \right)^3}{T^6} - 48 \left( \frac{t'}{T} \right)^3 \left( f_{L}(t') \right)^4, \]  

(24)

\[ S_1 = 2 \left( \frac{\rho_0}{\rho(z)} \right)^2 \left[ \frac{S_{12}}{\omega_0^2} - f_{s}(t') \right] Q_1 + \left( \frac{S_{13}}{\omega_0^3} - \frac{S_{11}}{\omega_0} \right) Q_2 \]  

(25)

\[ S_2 = \frac{4 z}{z_R} \left( \frac{\rho_0}{\rho(z)} \right)^3 \left[ \frac{2 S_{13}}{\omega_0^3} - \frac{S_{11}}{\omega_0} \right] Q_3 - 2 \frac{S_{12}}{\omega_0^2} Q_4, \]  

(26)

\[ S_3 = \frac{z^2}{z_R} \left( \frac{\rho_0}{\rho(z)} \right)^4 \left[ \frac{S_{12}}{\omega_0^2} Q_3 + \frac{S_{13}}{\omega_0^3} Q_4 \right], \]  

(27)

\[ S_{11} = \frac{f_{s}(t')}{T} \text{tanh} \left( \frac{t'}{T} \right), \]  

(28)

\[ S_{12} = \frac{f_{s}(t')}{T^2} \left[ 1 - 2 \left( f_{s}(t') \right)^2 \right], \]  

(29)

\[ S_{13} = \frac{f_{s}(t')}{T^3} \left[ 1 - 6 \left( f_{s}(t') \right)^2 \right] \]  

(30)
\( \rho_0 = \frac{1}{\sqrt{1 + \left(\frac{z}{z_R}\right)^2}} \), \( \rho(z) \) is beam spot size at beam waist and \( \rho_z \) is wave packet transverse dimension.

Gouy phase shift \( \approx \tan^{-1}\left(\frac{z}{z_R}\right) \).

And \( m = \left\lfloor \frac{1}{2} \right\rfloor \) where the parameter \( m \) measures space variation of the field at point \((r, \theta, z)\).

\[
3 \text{ Results and Discussion}
\]

For TM\(_{01}\) wave packets \( B_z = B_r = E_0 = 0 \)

\[
dz = \frac{v_z}{v}, \quad \frac{dv_z}{dt} = -\frac{q}{m} E_z(z,t) \left(1 - \frac{v_z^2}{c^2}\right)^{3/2} \quad \ldots(45)
\]

These equations are solved by using 5th order Runge-Kutta method. The calculation parameters are \( \lambda_0 = 0.8 \times 10^{-6} \text{m} \), \( \rho_0 = 10^{-8} \text{m} \), initial phase \( \phi_0 = -0.2\pi \).

The electron is initially at rest at beam waist. The intensity of electric field is defined as:

\[
I = \frac{(\rho_0 A)^2}{480\pi e} = 10^{22} \text{W/cm}^2.
\]

The longitudinal electric field of a paraxial and monochromatic TM\(_{01}\) beam experiences a Gouy phase shift of \( 2\tan^{-1}(z/z_R) \); and hence a phase variation of 0 to \( \pi \) will occur from \( z = 0 \) to \( \infty \). And electron initially at rest, at the beam waist will not gain the net kinetic energy in a complete cycle. As it is being accelerated in positive half cycle and decelerated in the next (negative) half cycle, a delay is necessary to bring the electron from rest to a relativistic velocity\(^{25} \)). Here the Gouy phase shift is the essential tool to evolve the required delay. As a consequence of that delay, the electron will slip into the next half cycle. During successive acceleration and deceleration cycles, as the acceleration period exceeds the deceleration one, the electron moves away from the beam waist.

Longitudinal fields have small amplitude, but since they extend infinitely along the beam axis, they contain an infinite amount of energy. If the electron reaches a position around \( Z_R \) and if the field intensity is high enough the electron could then be accelerated to a relativistic velocity.

Our results show that the energy of accelerated electrons depends upon pulse duration, intensity, initial phase and detuning length. When the pulse intensity exceeds \( 10^{21} \text{W/cm}^2 \), the kinetic energy transferred to the accelerated electron depends upon the absolute phase of the pulse carrier and shape of the temporal profile. At ultra-high intensity \( (10^{22} \text{W/cm}^2) \), an electron could be accelerated from rest to GeV energies within a few millimetres and in a few femto-seconds. With longer pulses, electrons are trapped in the front of the pulse before the field reaches its peak values. Thus, the longer pulses lead to lower values of peak energy gain. However, in case of the single-cycle pulses, some electrons can penetrate the field up to its maximum value before
being trapped and they are accelerated to greater energy.

If an ultra-intense ultra-short pulse with a TM<sub>01</sub> profile is used, the electrons that are close to the propagation axis are accelerated forward by the longitudinal electric field. Energy can be transferred to an electron initially at rest at the waist of an ultra-short TM<sub>01</sub> wave packet from the longitudinal electric field component. In this scheme we propose the particle acceleration in free space and hence the concept of ponderomotive acceleration is not relevant. Electrons at rest at the waist that are promptly accelerated to highly relativistic energies tend to progressively get out of phase with the field; they are decelerated in the last part of their trajectory and experience a small overall energy gain.

Figures 1[(a)-(c)] describe the usual response of kinetic energy gained by the electron at on-axis points for different pulse cycles. We find that for various temporal profiles of the laser pulse, while increasing the number of cycles, kinetic energy gain of electron decreases. However, in the case of single-cycle Lorentzian pulse the energy gain by the electron is ~ three times larger than number of cycles p = 2 or p = 3.

Figure 2 shows the response in kinetic energy gain of electron for double cycle at on-axis points for different temporal profiles. The results of our calculations show that for a given intensity of the laser pulse the magnitude of acceleration of electrons depends on the temporal profile. As an example, a laser with a Lorentzian temporal profile is preferable as comparison to Gaussian and hyperbolic secant profile pulse. The Lorentzian pulse lasts for a longer time and consequently imparts relatively higher magnitude of the acceleration. Figures 3[(a)-(c)] represent the variation in kinetic energy gain by the on-axis electron for single cycle Gaussian, Lorentzian and hyperbolic secant temporal profiles for different initial phases. At low intensity of the incident laser,
the electron moves with the carrier and it will not be trapped within a half cycle of pulse. At high intensity, the electron is trapped within a half cycle of the pulse in a few femto-seconds and few mm of longitudinal distance. In order to obtain the higher values of possible acceleration, our results show that a single cycle Lorentzian pulse profile is preferable.

Acknowledgement

Financial support from the Department of Science & Technology, New Delhi (Government of India) is thankfully acknowledged. The authors are thankful to Prof Naresh Dadhich, Vice-Chancellor, VMOU, Kota for his kind support to this project and also for his keen interest in the outcome of this work. Thanks are also due to Mr Rakesh Sharma, Director, S & T, VMOU, Kota for providing administrative support.

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