Angular speed-dependent performance of generalized irreversible Carnot engine

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On the basis of a generalized irreversible Carnot engine model with the losses of heat resistance, heat leakage and internal irreversibility, the angular speed-dependent performance of the engine is analyzed and optimized with Newton’s heat transfer law $Q \propto \Delta(T)$ between the working fluid and the heat reservoirs. The relations among the dimensionless power output, efficiency and the angular speed of the heat engine, the optimal angular speed corresponding to the maximum power output and the maximum efficiency are derived. Numerical examples show the effects of the heat leakage and the internal irreversibility on the angular speed-dependent performance of the engine. The results can provide guidelines for selecting appropriate working point of heat engines.

Keywords: Finite time thermodynamics, Heat engine, Generalized irreversible, Angular speed, Newton’s heat transfer law

1 Introduction
Since Novikov (1957), Chambadal (1957) and Curzon-Ahlborn (1975) introduced the process of heat transfer, and then derived the famous CA efficiency and established the theory of finite time thermodynamics (FTT), many authors have studied performance of various kinds of heat engines and obtained a large number of useful results$^{1-15}$. Spence and Harrison$^{16}$ analyzed the angular speed-dependent performance of an endoreversible Carnot engine with Newton’s heat transfer law $q \propto \Delta(T)$ between working fluid and heat reservoirs. Rebhan and Ahlborn$^{17}$ analyzed frequency-dependent performance of an endoreversible Carnot engine with Newton’s heat transfer law between the working fluid and the heat reservoirs. Qin et al.$^{18}$ analyzed frequency-dependent performance of an endoreversible Carnot engine with linear phenomenological heat transfer law $q \propto \Delta(T^{-1})$ between the working fluid and the heat reservoirs. Wang et al.$^{19}$ analyzed frequency-dependent performance of a generalized irreversible Carnot engine with linear phenomenological heat transfer law between working fluid and heat reservoirs. On the basis of Ref. 16 this paper will further analyze the influence of angular speed on the power and efficiency characteristic of a generalized irreversible Carnot engine with the losses of heat resistance, heat leakage and internal irreversibility$^{9,19-22}$ with Newton’s heat transfer law between the working fluid and the heat reservoirs.

2 Physical Model
Considering the model of a generalized irreversible reciprocating Carnot heat engine$^{9,19-22}$ as shown in Fig. 1, its working conditions are as follows:

(i) A reciprocating Carnot heat engine, each cycle of the working fluid consists of two isothermal and two adiabatic processes which are irreversible ones. Because of the heat resistance, the working fluid’s temperatures ($T_1$ and $T_2$) are different from the reservoirs’ temperatures ($T_H$ and $T_L$) and the four temperatures are of the following decreasing order: $T_H > T_1 > T_2 > T_L$.

(ii) There is a heat leakage from the heat source to the heat sink. Applying the Bejan’s model of the heat leakage$^{23}$, the amount of the heat leakage is

Fig. 1 — Model of a generalized irreversible Carnot heat engine
\(Q_i = q_i \tau = C_i (T_h - T_i) \tau\) where \(q_i\) is the rate of the heat leakage, \(C_i\) is the coefficient of the heat leakage and \(\tau\) is the cycle period. Assuming that the heat transferring from and to the high- and low-temperature heat exchangers are \(Q_1\) and \(Q_2\) respectively, and the heat transfer supplied by the heat source is \(Q_H\) and the heat transfer released to the heat sink is \(Q_L\), \(Q_H = Q_i + Q_1\) and \(Q_L = Q_2 + Q_2\).

(iii) A constant coefficient \(\phi\) is introduced to characterize the additional internal miscellaneous irreversibility effects: \(\phi = Q_2/Q_2' \geq 1\), where \(Q_2\) is the heat flow from the heat reservoir to the hot working fluid for the generalized irreversible Carnot heat engine and \(Q_2'\) is that for the heat engine with the losses of the heat resistance and the heat leakage.

This model will be reduced to the endoreversible Carnot engine model\(^{16,18,24}\) when \(C_i=0\) and \(\phi=1\). It will be reduced to the irreversible Carnot engine model with the losses of the heat resistance and the heat leakage\(^{23,25}\) when \(C_i>0\) and \(\phi=1\). It will be reduced to the irreversible Carnot engine model with the losses of the heat resistance and the internal irreversibility\(^{26}\) when \(C_i=0\) and \(\phi=1\). It is the generalized irreversible Carnot engine model\(^{9,19-22}\) when \(C_i>0\) and \(\phi=1\).

3 Angular Speed-Dependent Performance

Assuming that the heat transfer between the working fluid and heat reservoir obey Newton’s heat transfer law based on the model mentioned above, \(Q_i\), which is the heat flow from the heat source to the hot working fluid and \(Q_2\), which is the heat flow from the cold working fluid to the heat sink are

\[Q_i = \alpha (T_h - T_i) t_1\]  \hspace{1cm} (1)

\[Q_2 = \beta (T_2 - T_i) t_2\]  \hspace{1cm} (2)

respectively, where \(\alpha\) and \(\beta\) are the overall heat transfer coefficients of the high- and low-temperature-side heat exchangers, respectively, \(t_1\) and \(t_2\) are the times spent on the isothermal heat absorption and heat rejection processes, respectively. The cycle period is given by \(\tau = t_1 + t_2\). In fact, parameter \(\gamma\) has no effect on the final results in this paper, so \(\gamma = \eta t_1 + t_2\) at given value \(\gamma = 1\), and the angular speed becomes \(\omega = 2\pi/\tau\).

The second law of thermodynamics requires that

\[
\frac{Q_2}{\phi T_2} - \frac{Q_i}{T_i}.
\]  \hspace{1cm} (3)

Defining the heat transfer coefficient ratio and the time ratio as follows:

\[
\delta = \beta/\alpha, \rho = t_1/t_2 \hspace{1cm} (4)
\]

then,

\[
t_1 = 2\pi \rho / \omega (\rho + 1) \hspace{1cm} (5)
\]

Combining Eqs. (1)-(5) gives:

\[
\frac{T_2}{T_1} = \frac{2\pi \rho \beta T_1}{2\pi \alpha T_1 \delta - \omega (\rho + 1)(\phi \rho + \delta) Q_i} \hspace{1cm} (6)
\]

The general form of power output \(P(\omega)\) is:

\[
P(\omega) = W/\tau \hspace{1cm} (7)
\]

where \(W\) is the work output of the cycle. Combining \(W = Q_i - Q_2\), \(\tau = 2\pi/\omega\), Eqs. (3) and (7) gives:

\[
P(\omega) = \omega \left[ \frac{Q_i}{2\pi} - \frac{Q_2}{2\pi} \right] = \frac{\omega Q_i}{2\pi} \left[ 1 - \frac{Q_2}{Q_i} \right] = \frac{\omega Q_i}{2\pi} \left( 1 - \frac{\phi T_2}{T_i} \right) \hspace{1cm} (8)
\]

The first law of thermodynamics gives that the thermal efficiency \((\eta)\) of the cycle is:

\[
\eta = \frac{Q_H - Q_L}{Q_H} = \frac{Q_i - Q_2}{Q_i + Q_i} = \left[ 1 - \frac{\phi T_2}{T_i} \right] / (1 + Q_i / Q_i) \hspace{1cm} (9)
\]

Substituting Eq. (6) into Eqs (8) and (9) yields the expressions of the power output and efficiency as follows:

\[
P(\omega) = \frac{\omega Q_i \left[ 2\pi \rho \beta (T_h - \phi T_1) - \omega (\rho + 1)(\phi \rho + \delta) Q_i \right]}{2\pi \left[ 2\pi \rho \beta T_h - \omega (\rho + 1)(\phi \rho + \delta) Q_i \right]} \hspace{1cm} (10)
\]

\[
\eta(\omega) = \frac{2\pi \rho \beta (T_h - \phi T_1) - \omega (\rho + 1)(\phi \rho + \delta) Q_i}{2\pi \rho \beta T_h - \omega (\rho + 1)(\phi \rho + \delta) Q_i} \hspace{1cm} (11)
\]

\[
\times \left[ 1 + 2\pi C_i (T_h - T_i) / (Q_i \omega) \right]
\]

Taking the derivatives of \(P\) and \(\eta\) with respect to \(\sigma\) and setting them equal to zero (\(dP/d\sigma=0\) and \(d\eta/d\sigma=0\)) yields:
\( \rho = \sqrt{\delta / \phi} \) ... (12)

The corresponding optimal power output and optimal efficiency are

\[
Q_0(\omega) = \frac{2\pi \beta (T_H - T_L) \sqrt{\delta / \phi} - \omega (\sqrt{\delta / \phi} + 1)}{\left( \phi \sqrt{\delta / \phi} + \delta \right) Q_i} \] ...

\[
P(\omega) = \frac{2\pi T_H \sqrt{\delta / \phi} - \omega (\sqrt{\delta / \phi} + 1)}{\left( \phi \sqrt{\delta / \phi} + \delta \right) Q_i} \] ...

\[
\eta(\omega) = \frac{2\pi \beta (T_H - T_L) \sqrt{\delta / \phi} - \omega (\sqrt{\delta / \phi} + 1)}{\left( \phi \sqrt{\delta / \phi} + \delta \right) Q_i} \times \left[ 1 + 2\pi C_i (T_H - T_L) / (Q_i \omega) \right] \] ...

respectively. Based on the maximum angular speed of an endoreversible heat engine \(^{16}\):

\[
\omega_{\text{max}, \phi=1, C_i=0} = \frac{2\pi \beta (T_H - T_L) \sqrt{\delta / \phi}}{\left( \phi \sqrt{\delta / \phi} + \delta \right) Q_i} \] ...

defining the dimensionless angular speed as:

\[
\bar{\omega} = \omega / \omega_{\text{max}, \phi=1, C_i=0} \] ...

the power output and efficiency are:

\[
P(\omega) = \frac{\beta T_H \sqrt{\delta / \phi} (1-x) \bar{\omega}}{(1-x) \left( \phi \sqrt{\delta / \phi} + \delta \right) \bar{\omega} (1-x) / (\sqrt{\delta / \phi} + \delta)} \] ...

\[
\eta(\omega) = \frac{(1-x) - \bar{\omega} \left( \phi \sqrt{\delta / \phi} + \delta \right)(1-x) / (\sqrt{\delta / \phi} + \delta)}{1 - \bar{\omega} \left( \phi \sqrt{\delta / \phi} + \delta \right)(1-x) / (\sqrt{\delta / \phi} + \delta)} \times \left[ 1 + C_i / \alpha / \left( \phi \sqrt{\delta / \phi} + \delta \right) \right] \] ...

respectively, where

\[ x = T_L / T_H \] ...

The maximum power output of an endoreversible Carnot heat engine obtained in Ref. 24 is:

\[
P_{\text{max}, \phi=1, C_i=0} = \frac{\beta}{(1 + \sqrt{\delta})^2} \left( \sqrt{T_H} - \sqrt{T_L} \right)^2 \] ...

The maximum efficiency of an endoreversible Carnot heat engine is:

\[
\eta_{\text{max}, \phi=1, C_i=0} = 1 - T_L / T_H \] ...

Thus the dimensionless power output (\( \pi \)) and the dimensionless efficiency (\( \varepsilon \)) are:

\[
\pi = \frac{P(\omega)}{P_{\text{max}, \phi=1, C_i=0}} = \frac{\frac{(1-x)}{(\phi \sqrt{\delta / \phi} + \delta)}}{(1-x)(1-x)} \times \bar{\omega} \left( \phi \sqrt{\delta / \phi} + \delta \right)(1-x) / (\sqrt{\delta / \phi} + \delta) \] ...

\[
\varepsilon = \frac{\eta(\omega)}{\eta_{\text{max}, \phi=1, C_i=0}} = \frac{1 - \bar{\omega} \left( \phi \sqrt{\delta / \phi} + \delta \right)(1-x) / (\sqrt{\delta / \phi} + \delta)}{1 - (\phi \sqrt{\delta / \phi} + \delta)(1-x) / (\sqrt{\delta / \phi} + \delta)} \times \left[ 1 + C_i / \alpha / \left( \phi \sqrt{\delta / \phi} + \delta \right) \right] \] ...

respectively. Taking the derivative of \( \pi \) with respect to \( \bar{\omega} \) and setting it equal to zero (\( d\pi / d\bar{\omega} = 0 \)) yields the optimal dimensionless angular speed corresponding to the maximum power, the maximum dimensionless power and the corresponding dimensionless efficiency

\[ \bar{\omega}_{\text{opt}, p} = \frac{\left( \phi \sqrt{\delta / \phi} + \delta \right)(1-x)}{(1-x)} \] ...

\[ p_{\text{max}} = \frac{B(1-x)^2}{(1-x)^2} \] ...

\[ n_{\text{opt}, p} = \frac{(1-x)}{(1-x) / (\phi \sqrt{\delta / \phi} + \delta)} \] ...

... (24)

... (25)
\[ \varepsilon_p = \frac{\sqrt{\delta} / \phi \delta (1 - \sqrt{\phi x})^2}{(1 - x) \left[ \frac{\sqrt{\delta} / \phi \delta (1 - \sqrt{\phi x})}{1 - \sqrt{\phi x}} \right] + \frac{C}{\alpha} \left( \sqrt{\delta} / \phi + \delta \right) (1 - x)} \] ...

respectively, where

\[ B = \frac{\sqrt{\delta} / \phi (1 + \sqrt{\delta})^2}{(\phi \sqrt{\delta} / \phi + \delta)(\sqrt{\delta} / \phi + 1)} \] ...

Taking the derivative of \( \varepsilon \) with respect to \( \bar{\omega} \) and setting it equal to zero (\( d\varepsilon / d\bar{\omega} = 0 \)) yields:

\[ (a^2b - a\phi x)\bar{\omega}^2 - 2ab\bar{\omega} + b(1 - \phi x) = 0 \] ...

where

\[ a = \frac{(\phi \sqrt{\delta} / \phi + \delta)(1 - x)}{(\sqrt{\delta} / \phi + \delta)} \] ...

\[ b = \frac{C_i \left( \sqrt{\delta} / \phi + \delta \right) (\sqrt{\delta} / \phi + 1)}{\delta \alpha \sqrt{\delta} / \phi} \] ...

Thus, when \( ab \neq \phi x \), the optimal dimensionless angular speed corresponding to the maximum efficiency, the maximum dimensionless efficiency and the corresponding dimensionless power output are

\[ \bar{\omega}_{\text{opt}, \eta} = \frac{ab - \sqrt{ab(1 + ab - \phi x)\phi x}}{a(ab - \phi x)} \] ...

\[ \varepsilon_{\text{max}} = \frac{A(1 - \phi x - A)}{(1 - x)(A + ab)(1 - A)} \] ...

\[ \pi_{\eta} = \frac{(1 + \sqrt{x})(1 - \phi x - A)AC}{a(1 - \sqrt{x})(1 - A)} \] ...

respectively, where

\[ A = \frac{ab - \sqrt{ab(1 + ab - \phi x)\phi x}}{ab - \phi x} \] ...

\[ C = \frac{B(\phi \sqrt{\delta} / \phi + \delta)}{(\sqrt{\delta} / \phi + \delta)} \] ...

When \( ab = \phi x \), the corresponding results are

\[ \bar{\omega}_{\text{opt}, \eta} = \frac{1 - \phi x}{2a} \] ...

\[ \varepsilon_{\text{max}} = \frac{(1 - \phi x)^2}{(1 - x)(1 + \phi x)^2} \] ...

\[ \pi_{\eta} = \frac{C(1 + \sqrt{x})(1 - \phi x)^2}{2a(1 - \sqrt{x})(1 + \phi x)} \] ...

respectively.

4 Analysis and Discussion

(i) In this paper, the dimensionless efficiency and dimensionless power output versus dimensionless angular speed of the generalized irreversible Carnot heat engine are derived, the optimal dimensionless angular speed (\( \bar{\omega}_{\text{opt}, P} \)) corresponding to the maximum power output, the maximum dimensionless power (\( \pi_{\text{max}} \)), the corresponding efficiency (\( \varepsilon_p \)), the optimal dimensionless angular speed (\( \bar{\omega}_{\text{opt}, \eta} \)) corresponding to the maximum efficiency, the maximum dimensionless efficiency (\( \varepsilon_{\text{max}} \)) and the corresponding power \( \pi_{\eta} \) are obtained. The results show that there are effects of the heat leakage on the efficiency and \( \bar{\omega}_{\text{opt}, \eta} \), and no effects on the power output and \( \bar{\omega}_{\text{opt}, P} \).

(ii) If there is only heat resistance loss in the cycle (\( \phi = 1 \) and \( C_i = 0 \)) and if \( \delta = 1 \), Eqs. (22) and (23) are reduced to:

\[ \varepsilon = f(\omega)(1 - \bar{\omega}) \] ...

\[ \pi / (1 + \sqrt{x})^2 = \bar{\omega} \varepsilon \] ...

where

\[ f(\omega) = \frac{1}{1 - \bar{\omega}(1 - x)} \] ...

Ref. 16 only considered the time ratio \( \rho = 1 \). In fact, \( \rho = \bar{\omega} \) hold. \( \rho \) is optimized and thus \( \rho = \sqrt{\delta} \) is obtained in this paper. The condition is reduced to that of
Ref. 16 when \( \delta = 1 \). In addition, the optimal dimensionless angular speeds corresponding to the maximum power output and corresponding to the maximum efficiency are, respectively,

\[
\bar{\omega}_{\text{opt},P} = 1 / \left(1 + \sqrt{x}\right) \quad \text{and} \quad \bar{\omega}_{\text{opt},\eta} = 0.
\]

(iii) If there are only heat resistance and heat leakage losses in the cycle (\( \phi = 1 \) and \( C > 0 \)), the dimensionless power and dimensionless efficiency versus dimensionless angular speed are

\[
\pi = \frac{C \left(1 + \sqrt{x}\right)^2 \left(1 - \bar{\omega}\right)}{\left[1 - \bar{\omega}(1 - x)\right]} \quad \text{... (42)}
\]

\[
\epsilon = \frac{(1 - \phi x) / (1 - x) - \bar{\omega}}{\left[1 - (1 - x)\bar{\omega}\right] \left[1 + \frac{C \left(1 + \sqrt{x}\right) \sqrt{\delta / \phi + \delta}}{\left(\sqrt{\delta + 1}\right)^2}\right]} \quad \text{... (43)}
\]

respectively. And the optimal dimensionless angular speeds corresponding to the maximum power output and corresponding to the maximum efficiency are

\[
\bar{\omega}_{\text{opt},P} = 1 / \left(1 + \sqrt{x}\right) \quad \text{... (44)}
\]

\[
\bar{\omega}_{\text{opt},\eta} = \frac{a_i b - \sqrt{a_i b (1 + a_i b - x) x}}{a_i (a_i b - x)} \quad \text{... (45)}
\]

respectively, where \( a_i = 1 - x \) is the expression of \( a \) with \( \phi = 1 \).

(iv) If there is no heat leakage in the cycle (\( \phi > 1 \) and \( C = 0 \)), the dimensionless power and dimensionless efficiency versus dimensionless angular speed are, respectively,

\[
\pi = \frac{C \left(1 + \sqrt{x}\right)(1 - \phi x - a \bar{\omega}) \bar{\omega}}{\left(1 - \sqrt{x}\right)(1 - a \bar{\omega})} \quad \text{... (46)}
\]

\[
\epsilon = \frac{1 - \phi x - a \bar{\omega}}{(1 - x)(1 - a \bar{\omega})} \quad \text{... (47)}
\]

and the optimal dimensionless angular speeds corresponding to the maximum power output and corresponding to the maximum efficiency are:

\[
\bar{\omega}_{\text{opt},P} = \frac{\left(\sqrt{\delta / \phi + \delta}\right)(1 - \sqrt{\phi x})}{\left(\phi \sqrt{\delta / \phi + \delta}\right)(1 - x)} \quad \text{... (48)}
\]

\[
\bar{\omega}_{\text{opt},\eta} = 0 \quad \text{... (49)}
\]

respectively.

(v) Hence, the efficiency decreases as the accretion of the heat leakage loss which has an influence on the curve of \( \epsilon \) versus \( \bar{\omega} \) and no influence on the curve of \( \pi \) versus \( \bar{\omega} \). The power output and efficiency decrease as the accretion of the internal irreversible factor (\( \phi \)) which has an influence on the curve of \( \epsilon \) versus \( \bar{\omega} \) or the curve of \( \pi \) versus \( \bar{\omega} \). If there is no heat leakage loss, \( \bar{\omega}_{\text{opt},\eta} \) is equal to zero.

### 5 Numerical Examples

\( \alpha = \beta \) (\( \delta = 1 \)) and \( T_L / T_H = 7/9 \) (\( x = 7/9 \)) are set according to Ref. (6). The effect of the internal irreversible factor (\( \phi \)) and heat leakage on the dimensionless power output versus dimensionless angular speed characteristic is shown in Fig. 2. It shows that the heat leakage has no influence on the power output of the generalized irreversible Carnot heat engine and this conclusion is in accordance with the result obtained from Eq. (22). It also shows that the curves of the dimensionless power output versus the dimensionless angular speed are like parabola, that the maximum operating angular speed, the maximum power output of the heat engine and the optimal angular speed corresponding to the maximum power output decrease with the increase of internal irreversible factor. These conclusions are also presented in Table 1.
The dimensionless efficiency versus the dimensionless angular speed characteristic for fixed values of \( \phi = 1.0, 1.1, 1.2 \) and \( C/\alpha = 0, 0.04, 0.08 \) are shown in Fig. 3. It shows that the efficiency is affected by the internal irreversible factor and the heat leakage, the curves of the dimensionless efficiency versus dimensionless angular speed are like to beeline and the power output decreases with the increase of angular speed if there is no heat leakage \((C/\alpha = 0)\), and that the curves of dimensionless efficiency versus dimensionless angular speed are like parabola and change quantitatively and qualitatively if there exists heat leakage \((C/\alpha > 0)\). In addition, the maximum operating angular speed is not affected by the heat leakage, but decreases with the increase of \( \phi \) as shown in Fig. 2.

The influence of the internal irreversible factor \((\phi)\) and heat leakage on the optimal dimensionless angular speeds \((\bar{\omega}_{\text{opt}, P} \text{ and } \bar{\omega}_{\text{opt}, \eta})\) corresponding to the maximum power output and the maximum efficiency, respectively, are shown in Fig. 4. It shows that \( \bar{\omega}_{\text{opt}, \eta} \) is affected by the heat leakage and the internal irreversible factor and decreases with the decrease of \( C/\alpha \) or the increase of \( \phi \), that \( \bar{\omega}_{\text{opt}, P} \) is influenced by the internal irreversible factor but does not by the heat leakage, decreases with the increase of \( \phi \) and is constant at a given value of \( \phi \). In addition, \( \bar{\omega}_{\text{opt}, \eta} \) is less than \( \bar{\omega}_{\text{opt}, P} \) for a fixed \( \phi \). These conclusions are all shown directly in Figs 2 and 3 and Table 1. It is interesting that \( \bar{\omega}_{\text{opt}, \eta} \) is equal to \( \bar{\omega}_{\text{opt}, P} \) when the heat leakage loss is infinite, which is also proved by the fact that Eq. (31) is in accord with Eq. (24) if \( b \to \infty (C/\alpha \to \infty) \).

The maximum dimensionless power output \((\pi_{\text{max}})\), the optimal dimensionless angular speed \((\bar{\omega}_{\text{opt}, P})\) and the dimensionless efficiency \((\varepsilon_{P})\) corresponding to the maximum power output, the maximum dimensionless efficiency \((\varepsilon_{\text{max}})\), the optimal dimensionless angular speed \((\bar{\omega}_{\text{opt}, \eta})\) and the dimensionless power output \((\pi_{\eta})\) corresponding to the maximum efficiency are presented in Table 1.

### Table 1 — Results of key parameters

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<th>( \phi )</th>
<th>( 1.0 )</th>
<th>( 1.0 )</th>
<th>( 1.0 )</th>
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<th>( 1.1 )</th>
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<th>( 1.2 )</th>
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<td>0.08</td>
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<td>1.0000</td>
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<td>0.3850</td>
<td>0.3850</td>
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<td>0.0753</td>
<td>0.0753</td>
</tr>
<tr>
<td>( \bar{\omega}_{\text{opt}, P} )</td>
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<td>0.5314</td>
<td>0.5314</td>
<td>0.3220</td>
<td>0.3220</td>
<td>0.3220</td>
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<td>0.1393</td>
<td>0.1393</td>
</tr>
<tr>
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<td>0.2255</td>
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Fig. 3 — Dimensionless efficiency versus angular speed characteristic of the generalized irreversible Carnot heat engine

Fig. 4 — Effect of irreversible factor and heat leakage on optimal angular speed
6 Conclusions

On the basis of a generalized irreversible Carnot engine model, the angular speed-dependent performance of the engine is analyzed with Newton’s heat transfer law \( Q \propto \Delta(T) \) between working fluid and heat reservoirs. The relations among dimensionless power output (\( \pi \)), efficiency (\( \eta \)) of the heat engine, an optimal angular speed corresponding to maximum power output and that corresponding to maximum efficiency are derived. The corresponding curves are given, the influence features of the heat leakage and internal irreversible factor on the performance of the engine are analyzed by numerical calculations. There’s no influence of heat leakage on the power output of the generalized irreversible Carnot heat engine, however, the curve of the dimensionless angular speed changes qualitatively because of the existence of the heat leakage. In addition, \( \bar{\omega}_{\text{opt}, P} \) is not influenced by the heat leakage while \( \bar{\omega}_{\text{opt}, \eta} \) increases with the increase of heat leakage. \( \bar{\omega}_{\text{opt}, P} \) and \( \bar{\omega}_{\text{opt}, \eta} \) both decrease with the increase of \( \phi \). \( \bar{\omega}_{\text{opt}, P} \) is a constant for fixed \( \phi \) and greater than \( \bar{\omega}_{\text{opt}, \eta} \) for fixed \( \phi \).

The results obtained in this paper are general and different from the angular speed-dependent performance of an endoreversible Carnot heat engine. Ref. 6 only considered the time ratio \( \rho=1 \). In fact, \( \rho \neq 1 \) holds, \( \rho \) should be optimized and the result of Ref. 6 is supplemented and perfected in this paper. The results obtained herein may be used in the design of reciprocating heat engines, pumps, refrigerators and other reciprocating devices, and can provide guidelines for selecting appropriate working point of reciprocating devices in practice.

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Nomenclature

\( A, a, B, C \) Intermediate variable
\( C_L \) Coefficient of heat leakage (kJ/K)
\( P \) Power output (kW)
\( Q_L \) Amount of heat leakage (kJ)
\( Q_T \) Heat transfer supplied by heat source (kJ)
\( q_h \) Rate of heat leakage (kJ)
\( T_{1,2} \) Working fluid’s temperatures (K)
\( T_{H, T_L} \) Reservoirs’ temperatures (K)
\( t_{1,2} \) Times spent on the processes (s)
\( W \) Work output (kJ)
\( x \) Ratio of reservoir temperatures
\( \alpha, \beta \) Overall heat transfer coefficients (kW/K)
\( \gamma \) Time coefficient
\( \delta \) Ratio of overall heat transfer coefficients
\( \epsilon \) Dimensionless efficiency
\( \eta \) Thermal efficiency
\( \pi \) Dimensionless power
\( \rho \) Time ratio
\( \phi \) Internal irreversibility
\( \omega \) Angular speed (rad/s)
\( \bar{\omega} \) Dimensionless angular speed
\( \tau \) Cycle period (s)

Subscripts

\( \max \) Maximum value
\( \text{opt} \) Optimal value
\( P \) Corresponding to the maximum power
\( \eta \) Corresponding to the maximum efficiency

References