Symbolic math for computation of radiation shielding

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Radiation transport calculations for shielding studies in the field of accelerator technology often involve intensive numerical computations. Traditionally, radiation transport equation is solved using finite difference scheme or advanced finite element method with respect to specific initial and boundary conditions suitable for the geometry of the problem. All these computations need CPU intensive computer codes for accurate calculation of scalar and angular fluxes. Computation using symbols of the analytical expression representing the transport equation as objects is an enhanced numerical technique in which the computation is completely algorithm and data oriented. Algorithm on the basis of symbolic math architecture is developed using Symbolic math toolbox of MATLAB software. Present paper describes the symbolic math algorithm and its application as a case study in which shielding calculation of rectangular slab geometry is studied for a line source of specific activity. Study of application of symbolic math in this domain evolves a new paradigm compared to the existing computer code such as DORT.

Keywords: Radiation transport, Symbolic math, Shielding

1 Introduction

Shielding evaluation is one of the prime tasks in radiation protection. Traditional approach of shielding analysis for a facility generating man made induced radiation is to use standard computer code such as QUADCG-GP [QUADCG manual1] and MCNP-4B [MCNP manual2]. The code QUADCG-GP uses point kernel method and the code MCNP-4B uses statistical method. However, Monte Carlo calculations require considerable amount of computation time and this code is not of use for bulk shielding analysis or deep penetration problem3. Further, computation burden increases rapidly for complex geometries including multiple sources. At the design time, shielding analysis is carried out using these codes, but a handy calculation of the shielding analysis is required at the time of operation of a specified facility. Analytically, expression of the point kernel method for an extended source is useful for such kind of shielding calculation. But, analytical expression contains some non-linear functions such as exponential function, Siveret integral and error function4. Analytical computation of all these functions is not possible. Instead an approximation method is searched for. Generally, standard numerical techniques4 are adopted for computing these functions. Number crunching methodologies, which are the backbone of all these numerical techniques lie under approximation. Therefore, fast, accurate and user friendly method is looked for computing these functions. Symbolic math provides such facilities of computing these kinds of functions [Symbolic math toolbox5].

The present paper describes the usage of symbolic math for computation of thickness of shielding material. Line source of finite length and plane parallel slab shield geometry are used to demonstrate the usage of the symbolic math. The advantage of symbolic math over the traditional numerical techniques is also discussed for shielding analysis. Symbolic MATH Toolbox of MATLAB 7.1 is used for the computation.

2 Symbolic Math

Symbolic Math is the mathematical methods in which computations are based on symbolic (objects) which carry the kernel of the computing engine. Kernels are basically designed by using standard numerical procedures coupled with variable precision arithmetic technique. Symbolic math is similar to object oriented programming and the corresponding toolbox (SYMBOLIC-MATH) of MATLAB 7.1 package is used for performing such type of computation. Many of the symbolic functions are MAPLE functions. Maple is based around a small kernel, written in C, which provides the Maple language. Most functionality is provided by libraries, which come from a variety of sources. Many numerical computations are performed by the NAG
Numerical Libraries, ATLAS libraries, or GMP libraries. Different functionality in Maple requires numerical data in different formats. Symbolic expressions are stored in memory as directed acyclic graphs.

3 Methodology of Shielding Evaluation

3.1 Point kernel technique

Point kernel technique is the analytical approximation, in which an extended source is divided into small number of point sources. Contribution of each of these point sources (dose rate) at the specified target point beyond the shield (plane parallel slab geometry) is summed up to have the resultant dose rate. The point kernel method encounters an account for scattered radiation which is usually implemented through semi-empirical approximation. Additional ‘build-up’ factor is introduced as a multiplier to the attenuated dose. Determination of the appropriate buildup factor can be rather complex as it depends upon the energy, thickness and type of material. Uncertainties in determining build-up factor can essentially limit the accuracy of point-kernel method. The point kernel is represented by $G(|r'-r|)$ which is defined as the response of a detector at the space point $r$ arising from a point source of radiation located at the space point $r'$.

3.2 Mathematical formulation of shielding evaluation

Based on the point kernel method, the uncollided flux of photons due to a line source at a point on the opposite surface of slab geometry of thickness $'x'$ is given by:

$$\varphi = \frac{V_s}{2\pi ha} \sec(\theta, \mu x)$$

where, $V_s$= total source strength in particles per second, $h= $ height of the source (cm), $a= $ distance from the axis of the cylinder to the point of interest (cm), sec(\theta, \mu x)= sec ant integral function, $\mu x= $ attenuating thickness of the shield material and $\theta= = \tan^{-1} (h/2a)$. Incorporating the Taylor’s build-up factor formula $B(\mu x)=A_1\exp(-A_2 \mu x)+A_3\exp(-A_3 \mu x)$ in Eq. (1), and using the dose conversion factor, $f_c=1.602x10^{-8}E_{\text{max}} (\mu_d/\rho)$ (rem/s per particles/s), where $(\mu_d/\rho)$ is mass absorption coefficient (cm$^2$/g) and $E_{\text{max}}$ is the photon energy (MeV); we have the dose rate at the target point.

$$D = \frac{V_s}{2\pi ha} f_c[A_i \sec (\theta, 1 + A_2 \mu x)]$$

$$+ A_4 \sec (\theta, 1 + A_4 \mu x)] \quad \text{(2)}$$

where $A_i$ ($i = 1,2,3,4$) are the Taylor’s constant and $A_4=1-A_1$.

Finally, the function to be zeroed for evaluating the shielding thickness which can be written as:

$$f(x)=2\pi haD_{\text{max}}-f_cV_s[A_1\sec (\theta(x), 1+A_2\mu x)]$$

$$+ A_4\sec (\theta(x), 1+A_3\mu x)] \quad \text{(3)}$$

where, $\theta(x)=\tan^{-1}\left(\frac{h}{2a}\right)=\tan^{-1}\left(\frac{h}{2(R + x)}\right)$, and $R$ is the radius of the line source.

3.3 Evaluation of the Sievert integral using symbolic math

For small and intermediate positive values of $'x'$ we have used the following approximation$^4$:

$$\sec(\theta, x) = \int_0^{\pi/2} e^{-x\sec(\varphi)} d\varphi$$

$$-\sum_{k=0}^{\infty} (\cos \theta)^{2k+1} E_{2k+1} \left(\frac{x}{\cos \theta}\right)$$

For $0<\theta<\pi/2$

where, $\alpha_0=1$ and $\alpha_k=[1,3,5, \ldots (2k-1)]/[2,4,6, \ldots (2k)]$ and $E_{2k+1}(x/\cos \theta)$ is the exponential integral function, the first part of the sievert integral is solved using first order Bickley function given in Eq.(4) and its series expansion is given in Eq. (5).

$$K_i(x) = \int_0^{\pi/2} e^{-x\sec(\varphi)} d\varphi$$

$$\gamma + \ln \left(\frac{x}{2}\right)\sum_{k=0}^{\infty} \frac{(\gamma + \ln \left(\frac{x}{2}\right))^k}{(k!)^2(2k+1)}$$

$$-x\sum_{k=0}^{\infty} \frac{(x^2/4)^k}{(k!)^2(2k+1)^2} - x^2\sum_{k=0}^{\infty} \frac{(x^2/4)^k}{(k!)^3(2k+1)}$$

where, $\gamma=0.5772156$ is the Euler constant and

$$\varphi(k+1) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}$$

And for large values of $'x'$, $\sec(\theta, x)$ is related to the Error function, erf(u) as:
The error function and the exponential integral functions were evaluated using symbolic math.

4 Results and Discussion
Schematic diagram of the code written for shielding calculations is shown in Fig. 1. The computed numerical values of Sievert integral and the Bickley function for various values of 'x' are shown in Fig. 2. The shielding calculation for a bare irradiated fuel element with nominal power of 1MW using 1500 Ci Co-60 as source for irradiation, $E_{\text{max}} = 1.65$ MeV; is carried out using line source assumption. Here, following values of the parameters were used: linear attenuation coefficient = 0.5500 cm$^{-1}$; $(\mu_{\text{tr}}/\rho) = 0.0274$ cm$^2$/g; Taylor’s build up factor parameters are $A_1 = 2.55$; $A_2 = 0.0260$; $A_3 = 0.1290$; Height of the rod = 30.50 cm; radius = 1.145 cm; Lead is used as shielding material and its geometry is considered as plane parallel slab, the shielding thickness obtained from calculation is 36.26 cm. Symbolic math provides the ease of computation and integrity part is intact as symbols behave as object, where procedures (algorithms) are encapsulated with the data.

References
5 Symbolic Math Toolbox of MATLAB 7.1.