Indian Journal of Engineering & Materials Sciences
Vol. 15, February 2008, pp. 14-22

Determination of fracture parameters of concrete based on water-cement ratio

Ragip Ince* & Kürşat Esat Alyamaç

Firat University, Engineering Faculty, Civil Engineering Department, Elazig, Turkey

Received 2 March 2007; accepted 5 February 2008

The aim of this study is to predict formulas based on the Abrams' Law between fracture parameters of concrete and w/c. Therefore, a series of three-point bend test on specimens were performed. The beams were made from mixes with water/cement ratios varying in the range of 0.34-0.85. The fracture parameters for each mix were calculated according to a popular fracture mechanics approach — two-parameter model. The present experimental data indicate that the fracture parameters of two-parameter model are inversely proportional to w/c ratios. In conclusion, the present test results are in agreement with the Abrams' Law.

Applications of Linear Elastic Fracture Mechanics (LEFM) to concrete were initiated by Kaplan[1] in 1961. In 1970s, experimental investigations on fracture mechanics of cementitious materials have indicated that LEFM was no longer valid for quasi-brittle materials such as concrete[2]. This inapplicability of LEFM is stemming from the existence of a relatively large inelastic zone in front of and around the tip of the main cracks in concrete. This so-called fracture process zone is ignored by LEFM. Consequently, several investigators have developed non-linear fracture-mechanics approaches to describe fracture-dominated failure of concrete structures[3-9].

These approaches primarily involve the fictitious crack model[3], the crack band model[4], the two-parameter model[5] (TPM), the effective crack model[6], the size effect model[7], the peak load method[8] and the variable-notch one-size test method[9]. Contrary to LEFM, in which a single fracture parameter is used such as the critical stress intensity factor, these models need at least two experimentally determined fracture parameters to characterize failure of concrete structures. Accordingly, they require either a lot of tests (at least three)[7-9] or a closed-loop testing system[3,5]. Analysis of any existing structure is impossible according to fracture mechanics for many approaches stated above, even if possible, specimens cored from structures must be tested after processed to a specific geometry[5,8,9]. In addition, several regression formulas[4,10-12] have been proposed to predict fracture parameters of concrete. Nevertheless, they give a little information about concrete mix proportion in order to design a concrete structure according to fracture mechanics principles.

There are many methods for concrete mix design according to compressive strength. The well-known Abrams' Law[13], which is an equation correlating the relationship between the strength and water/cement ratio, is a very important tool for determining a concrete mix proportion.

In this study, fifteen series of three-point bend tests on specimens were performed to predict formulas based on the Abrams' Law between fracture parameters of concrete and water-to-cement ratios. The beams were made from mixes with water/cement ratios ranging from 0.34 to 0.85. The fracture parameters for each batch were calculated according to TPM by using the peak load method. Finally, it has been concluded that there were quite strong relations between the fracture parameters of concrete and the water-cement ratio based on Abrams' Law.

Fracture Mechanics of Concrete

To analyze a concrete structure according to fracture mechanics, fracture parameters of the cementitious material must be determined at first. The studies on determining the fracture parameters of concrete were initiated by Kaplan[1]. He used the principles of classical linear elastic fracture mechanics (LEFM), which proposes a unique parameter (the critical stress intensity factor K_{Ic} or the critical strain energy release rate G_{Ic}) for concrete fracture. However, the subsequent experiments revealed that LEFM is not valid for concrete since K_{Ic}

* For correspondence (E-mail: rince@firat.edu.tr)

or G_{Ic} depends on size and geometry. The inapplicability of LEFM is because of the existence of an inelastic zone named fracture process zone (FPZ) in front of the crack in concrete. Several non-linear fracture mechanics models have been developed to characterize FPZ.

These models can be classified as the cohesive crack models (the fictitious crack model[3] and the crack band model[4]) and the effective crack models [the two parameters model (TPM)[5], the effective crack model[6] and the size effect model[7]]. The cohesive crack models simulate FPZ by a closing pressure, which diminishes near the crack tip while the effective crack models simulate FPZ by an effective crack length. The primary aim of these approaches is to determine the critical crack extension (size of FPZ) at the peak load $\Delta a = a_c - a_0$, in which a_c and a_0 are the critical crack length at the peak load and the initial crack length respectively. Nevertheless, a_c depends on the structural size, because it decreases as the size increases. Consequently, the non-linear fracture approaches propose that at least two fracture parameters are required for concrete fracture. However, the results of any fracture model can be easily adapted to the other fracture models of concrete.

Two-parameter model

A concrete structure fails, according to TPM, when the stress intensity factor K_I and the crack opening displacement *CTOD* reach their critical values, K_{Ic}^s and $CTOD_c$. These fracture parameters can be calculated by means of the following LEFM equations:

$$K_{Ic}^s = \sigma_{Nc}\sqrt{\pi a_c}\, f_1(\alpha_c) \quad \ldots(1)$$

$$CTOD_c = \frac{4\sigma_{Nc} a_c}{E_c} f_2(\alpha_c)\, f_3(\alpha_c, \beta) \quad \ldots(2)$$

in which σ_{Nc} is the nominal failure stress, d is the structure size, E_c is the Young's modulus, $\alpha_c = a_c/d$, $\beta = a_c/a_0$ and f_1, f_2, f_3 are the dimensionless functions, which depend on the geometry of the structure and on the load type. Since these functions can be found from any LEFM handbook[14], TPM can be easily used in structural analysis. Functions of f_1, f_2, f_3 for three-point bending specimen with span/depth ratio = 2.5 are[15]:

$$f_1(\alpha_c) = \frac{1.83 - 1.65\alpha_c + 4.76\alpha_c^2 - 5.3\alpha_c^3 + 2.51\alpha_c^4}{(1+2\alpha_c)(1-\alpha_c)^{3/2}} \quad \ldots(3)$$

$$f_2(\alpha_c) = 0.65 - 1.88\alpha_c + 3.02\alpha_c^2 - 2.69\alpha_c^3 + \frac{0.68}{(1-\alpha_c)^2} \quad \ldots(4)$$

$$f_3(\alpha_c, \beta) = \sqrt{(1-\beta)^2 + (1.081 - 1.149\alpha_c)(\beta - \beta^2)} \quad \ldots(5)$$

In this approach, the fracture parameters may be deduced from one of two different experimental methods: namely the compliance proposed by RILEM[16], and the peak load by Tang *et al*.[8]. In the first method, these parameters are determined from the relationship between load and crack mouth opening displacement (P-CMOD) of three point bending specimens with a central edge notch by using a closed-loop test equipment, as shown schematically in Fig. 1. The critical crack length a_c is calculated from two values taken from the P-CMOD curve: the

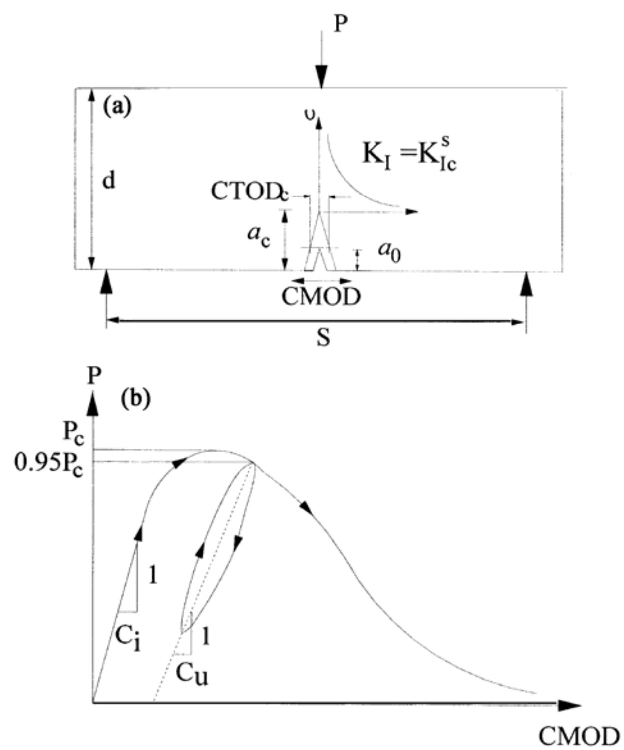

Fig. 1 — Determination of fracture parameters of concrete according to TPM, (a) characteristic values and (b) typical P-CMOD curve

initial compliance C_i and the unloading compliance C_u measured at about 95% of the peak load in the descending branch (Fig. 1b) as given by

$$a_c = a_0 \frac{C_u f_2(\alpha_0)}{C_i f_2(\alpha_c)} \qquad \ldots(6)$$

where $\alpha_0 = a_0/d$. Also, Young's modulus of concrete E_c can be also calculated using the initial compliance C_i or the unloading compliance C_u.

The peak-load method is a more simple method than the one introduced by RILEM in determining the fracture parameters of TPM because it requires uncomplicated testing equipment. However, it necessitates three or more distinct specimens due to the randomness of concrete properties. This is true for both methods. These specimens may be identical in size but different in initial crack length or have initial cracks of the same length but different sizes. For each of the tested specimen, the following equations can be written according to TPM:

$$K_I^i\left(\sigma_{Nc}^i,\ a_c^i\right) = K_{Ic}^s, \quad i = 1,2$$
$$CTOD^i\left(\sigma_{Nc}^i,\ a_c^i\right) = CTOD_c \qquad \ldots(7)$$

in which i denotes the i^{th} specimen. Consequently, the fracture parameters can be found by solving four simultaneous non-linear equations. However, three or more distinct specimens need to be tested for statistically valid results because random errors always exist in measured values of σ_{Nc}^1 and σ_{Nc}^2. In this case, a procedure based on statistic is used for calculating K_{Ic}^s and $CTOD_c$. In order to obtain this statistical adequacy, totally six specimens, with three different initial crack length and two specimens from each initial crack length, are sufficient in practice[8,15].

To determine the fracture parameters of concrete, the fracture test specimens such as three-point bend beams, the notched split tension cylinders, the holed split-tension cylinders and the eccentric compression prism can be used in the peak-load method while only three-point bend beams are utilized in the method by RILEM. In the peak-load method, the use of the cylindrical specimens, which can also be taken from the existing structures by core drilling, provides with great advantage for estimating fracture properties of existing structures based on fracture mechanics.

Fracture parameters of TPM, which are determined experimentally, can also be predicted by means of the following empirical equations based on a regression analysis[10]

$$K_{Ic}^s = 0.06\left(f_c'\right)^{0.75} \qquad \ldots(8)$$

$$CTOD_c = 0.00602\left(f_c'\right)^{0.13} \qquad \ldots(9)$$

where f_c' is the compressive strength of concrete. In these empirical equations f_c', K_{Ic}^s and $CTOD_c$ are in MPa, MPa√m and mm respectively. It is noted that these empirical equations only consider dependency of K_{Ic}^s and $CTOD_c$ on strength.

In practice, it is possible to combine the fracture parameters K_{Ic}^s and $CTOD_c$, and the material parameter E_c into a single length parameter Q. This is referred to as the brittleness number by Jenq and Shah[5] as given by Eq. (10):

$$Q = \left[\frac{E_c \,.CTOD_c}{K_{Ic}^s}\right]^2 \qquad \ldots(10)$$

Experimental studies have illustrated that the values of Q are in the range from 50-150 mm for mortar, and 150-350 mm for normal concrete.

Nevertheless, TPM, the effective crack model by Nallathambi and Karihaloo[6] and the size effect model by Bazant and Kazemi[7] give essentially equivalent results. The experimental studies revealed that the parameter of the critical stress intensity factor (K_{Ic}) is reasonably well correlated both in TPM and in the effective crack model[17]. In addition, value of K_{Ic}^s obtained by TPM can be transformed to the fracture energy in size effect model according to the well-known LEFM relation:

$$G_f = \left(K_{Ic}^s\right)^2 / E_c' \qquad \ldots(11)$$

in which for plain strain $E_c' = E_c/\left(1-\nu^2\right)$ for plain stress $E_c' = E_c$, ν = Poisson ratio. Similarly, $CTOD_c$ parameter can be transformed to the effective fracture process zone length (c_f) in size effect model parameter as given by Eq. (12)[12]:

$$c_f = \frac{\pi E_c'}{32 G_f} CTOD_c^2 \qquad \ldots(12)$$

The fracture energy parameter G_F determined by the fictitious crack model corresponds to the area under the entire stress-separation curve. This parameter is not identical to G_f in the size effect model. Statistical investigations by Bazant and Becq-Giraudon[12] have shown that the ratio G_F/G_f is about 2.5.

Experimental Studies

Experimental studies have shown that fracture parameters of concrete are particularly influenced by the four material parameters: compressive strength (f_c'), maximum aggregate size (d_{max}), water-cement ratio (w/c) and aggregate type[12,18]. On the other hand Bharatkumar *et al.*[19] investigated the effect of fly ash and slag on the fracture characteristics of high performance concrete. Consequently, they have pointed out that there is a reduction in the fracture energy of cementitious materials with addition of flay ash or slag. Ince[18] applied artificial neural networks, which is a very powerful tool to solve many civil engineering problems, in order to predict the fracture parameters K_{Ic}^s and $CTOD_c$ from w/c, d_{max} and f_c'. It was concluded from Fig. 2 that to make a distinction between mortar and concrete will be more realistic than considering a correlation between d_{max} and K_{Ic}^s when investigating the effect of d_{max} on K_{Ic}^s. For this reason, plain concrete with the maximum size of coarse aggregate 16 and 31.5 mm was only studied in this study.

The test specimen was a prism. Specimens 150 × 150 × 450 mm (span length = 380 mm) were cast in steel moulds. Fifteen series of specimens (90 prism specimens), namely C1-C15, were tested in different water-cement ratios ranging from 0.34 to 0.85. Although typical values of w/c ratios for plain concrete are in the range 0.4-0.6, these values are extended about ±0.05 here in order to find general expressions based on water/cement ratio. Additionally, w/c ratio in batch C7 was taken as 0.85 to produce low strength concrete. Six three-point bending specimens in each series were cast from the same batch of concrete. The specimens were cast as the notch face is at the bottom. The six prism specimens were classified into three groups of according to the relative initial crack length a_0/d = 0.1, 0.2 and 0.25. Three identical cylinder specimens with diameter 150 mm and 300 mm length were also cast from each batch of concrete to determine compressive strength of concrete. The prisms were filed to mould in two layers and the cylinder specimens in three layers. Each layer of concrete was compacted by not less than 25 strokes of a 16 mm diameter bullet-nosed rod. The maximum aggregate size was 31.5 mm for batch C1-C7 and 16 mm for C8-C15. The maximum sand grain size was 4 mm for each batch. Mineralogically, the aggregate was consisted of river. The aggregate and sand were air-dried prior to mixing. The Portland cement was used for the production of concrete mixtures that had a 28 day minimum compressive strength of 32.5 MPa. The Batches C1, C2, C8 and C9 required the use of a superplasticizer in order to increase their workability. All the specimens and cylinders were removed from the mould after 1 day and were subsequently cured till tested at 28[th] day in a moist room of 95% relative humidity and of about 23°C temperatures.

All the specimens were tested in a testing machine with a capacity of 2500 kN. The specimens were loaded monotonically until failure and care was taken to apply a constant loading rate. Typically, it took about 8 min (± 30 s) to reach the maximum load for each specimen. The identical cylinders were tested at an age same as the prism specimens. For each of the 90 specimens, Table 1 summarizes the water to cement ratio w/c, the maximum aggregate size d_{max}, the compressive strength f_c' of concrete, the initial crack length a_0 and the observed maximum load P_c.

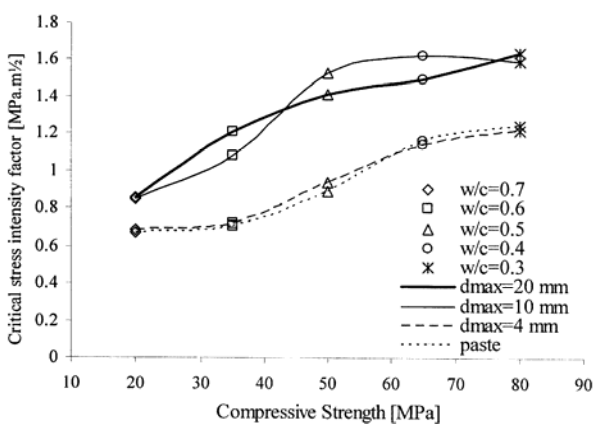

Fig. 2 — Effect of f_c' and d_{max} on K_{Ic}^s [18]

Analysis of Test Results

The following procedures based on statistics were used to evaluate parameters of TPM K_{Ic}^s and $CTOD_c$ according to the peak load method. Beams were grouped with reference to their initial crack length a_0 in Table 1. The K_{Ic}^s - $CTOD_c$ relationship were

Table 1 — Test results

Batch	w/c	d_{max} mm	f'_c MPa	a_0, mm						P_c (kN)					
				1	2	3	4	5	6	1	2	3	4	5	6
C1	0.34	31.5	53.68	15.8	15.8	30.0	31.0	37.0	37.0	25.2	24.6	21.7	20.5	17.9	17.3
C2	0.35	31.5	54.87	16.0	16.8	30.5	30.5	37.5	38.0	26.4	24.4	22.5	21.7	19.8	17.2
C3	0.42	31.5	27.92	14.5	13.3	19.1	28.8	37.0	36.4	15.5	12.4	13.0	12.8	11.0	10.1
C4	0.47	31.5	39.4	13.3	13.5	28.9	28.5	36.1	38.0	16.3	17.4	13.2	13.8	11.9	12.9
C5	0.49	31.5	24.35	16.5	16.5	32.0	31.5	38.0	38.5	17.0	16.8	13.7	15.9	11.8	12.1
C6	0.54	31.5	26.47	19.0	19.0	32.0	32.0	36.0	40.0	12.5	10.5	10.1	8.9	9.2	7.8
C7	0.85	31.5	7.39	16.0	16.5	31.5	31.0	38.0	38.5	3.5	4.1	2.8	3.3	2.6	2.9
C8	0.36	16	44.3	17.0	16.0	33.0	32.0	39.0	38.0	19.6	22.4	15.6	18.8	14.4	16.7
C9	0.37	16	41.79	16.0	16.0	32.0	31.0	39.0	38.0	21.2	20.4	18.4	17.1	14.8	15.2
C10	0.4	16	35.73	15.5	16.5	31.0	32.2	38.0	38.0	20.5	18.9	17.5	17.0	14.7	14.3
C11	0.44	16	32.88	14.0	13.4	28.5	28.6	36.2	35.9	13.6	15.7	12.5	13.3	10.5	12.6
C12	0.49	16	33.56	15.0	15.8	30.5	31.0	37.0	37.2	20.0	19.0	17.2	15.8	14.5	11.9
C13	0.5	16	35.72	18.3	13.4	29.8	28.7	36.9	36.5	15.8	20.3	14.0	17.0	12.4	14.3
C14	0.52	16	22.54	17.0	17.1	33.1	33.1	39.0	40.0	10.4	10.0	8.9	9.0	7.6	6.7
C15	0.66	16	15.24	16.0	16.5	31.0	32.0	38.5	39.5	9.5	9.1	8.0	8.5	7.7	7.2

determined for each group by utilizing Eqs (1)-(5), for instance for the batch C5 as shown in Fig. 3. In this paper, Young's modulus of concrete in Eq. (2) was determined according to ACI-318[20] as follows:

$$E_C = 4730\sqrt{f'_c} \qquad \ldots(13)$$

where E_c and f'_c are in MPa. As shown in Fig. 3, the three curves do not intersect at the same point owing to the randomness of the concrete properties and measurement errors. Thus, the sample standard deviation of values of K^s_{Ic}, at the same value of $CTOD_c$, must be calculated in the peak load method for all three K^s_{Ic} - $CTOD_c$ curves. Consequently, the average K^s_{Ic} - $CTOD_c$ curve was obtained for these three groups, as represented by the curved solid line in Fig 3. Then the standard deviation of groups was calculated as follows:

$$s(CTOD_c) = \sqrt{\frac{\sum_{i=1}^{n}\left(\overline{K^s_{Ic}} - K^s_{Ici}\right)^2}{n-1}} \qquad \ldots(14)$$

in which n is the number of groups (n = 3 in this study), $\overline{K^s_{Ic}}$ is the average value of K^s_{Ic} for all groups and K^s_{Ici} is the value of K^s_{Ic} for the i^{th} group. For example, K^s_{Ic} and $CTOD_c$ parameters have been calculated as 31.97 MPa√mm and 0.02343 mm at minimum value for s (s_{min}), for the batch C5 (Fig. 3). Table 2 summarizes the d_{max}, w/c, K^s_{Ic}, $CTOD_c$ and Q values calculated via Eq. (10) for each of the 15 batches. The mean Q values are 304.8 mm for d_{max} = 16 mm and 329.4 mm for d_{max} = 31.5 mm as shown in Table 2. Since the smaller values of Q point out a more brittle material behaviour, these values are in good agreement with the available findings in the literature.

Derivation of relationships between the fracture parameters and the water-cement ratio

One of the most important parameters influencing the behaviour of the concrete is water-cement ratio used in a concrete mix. Basically, concrete made with high w/c ratios will be low properties such as compression strength, Young's modulus and impermeability. When considering the compressive strength, this relation was first obtained by Abrams[13] in 1918. The so-called Abrams' Law is expressed as

$$f_c = \frac{K_1}{K_2^{1.5w/c}} \qquad \ldots(15)$$

where f_c is the compressive strength at fixed age, w/c is by weight and bulk density of cement is taken as

Table 2 — Fracture parameters of various mixes

Batch	d_{max} mm	w/c	K_{Ic}^{s} MPa√m	$CTOD_c$ μm	Q mm
C1	31.5	0.34	47.04	24.25	319
C2	31.5	0.35	49.4	25.3	322
C3	31.5	0.42	26.84	19.56	332
C4	31.5	0.47	31.24	19.04	327
C5	31.5	0.49	31.97	23.43	293
C6	31.5	0.54	23.86	19.04	377
C7	31.5	0.85	7.33	10.45	336
C8	16	0.36	40.95	23.42	324
C9	16	0.37	40.47	23.84	324
C10	16	0.4	38.13	23.61	306
C11	16	0.44	28.21	18.46	315
C12	16	0.49	33.96	19.87	257
C13	16	0.5	35.58	23.56	350
C14	16	0.52	19.4	14.55	284
C15	16	0.66	18.15	16.36	277

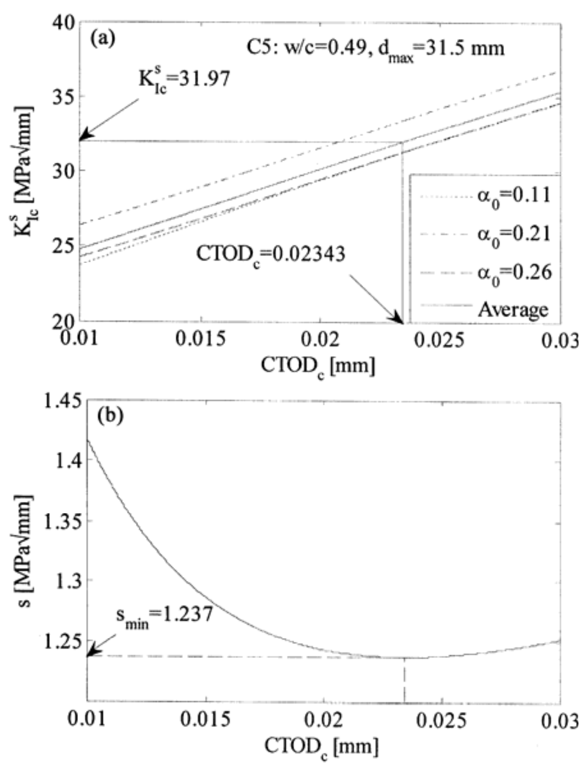

Fig. 3 — K_{Ic}^{s} versus $CTOD_c$ curve and s versus $CTOD_c$ curve for batch C5

1.5 t/m^3. K_1 and K_2 in Eq. (15) are the empirical constants, which depends usually on curing, test conditions, test age and cement properties. These constants can be calculated as $K_1 = A$ and $K_2 = \exp(-C/1.5)$ from the exponential regression made on $Y = Ae^{CX}$ with $Y = f_c$, X = w/c.

Figure 4 shows the results of the regression analysis according to Eq. (15) applied for batches C1-C15 in Table 1. It is derived from Fig. 4 that the test results are very close to the Abrams' Law because of the determination coefficient R^2 = 0.91. However, if single-sized aggregates were utilized in the mix design, Abrams' Law can be characterized by R^2 = 0.99[21]. Nevertheless, it is impossible in practice.

As already mentioned above, fracture parameters of concrete are particularly influenced by the four material parameters f_c', d_{max}, w/c and aggregate type. In this study, only rounded aggregates were used in the mix design. In addition, there are highly correlation between f_c' and w/c according to Eq. (15). Therefore, analysis of variance (ANOVA) tests can be used to test whether different combinations of the remaining material parameters d_{max} and w/c have an effect on the fracture parameters K_{Ic}^{s} and $CTOD_c$ or not. In ANOVA tests, w/c ratios have three levels: <0.4 (low), 0.4-0.6 (normal) and >0.6 (high) as denoted by 1, 2 and 3, in Table 3. The levels of d_{max} were chosen as 1 for 31.5 mm and 2 for 16 mm. A natural logarithmic transformation was performed on the parameters of K_{Ic}^{s} and $CTOD_c$ in Table 3 because Eq. (15) can be written as

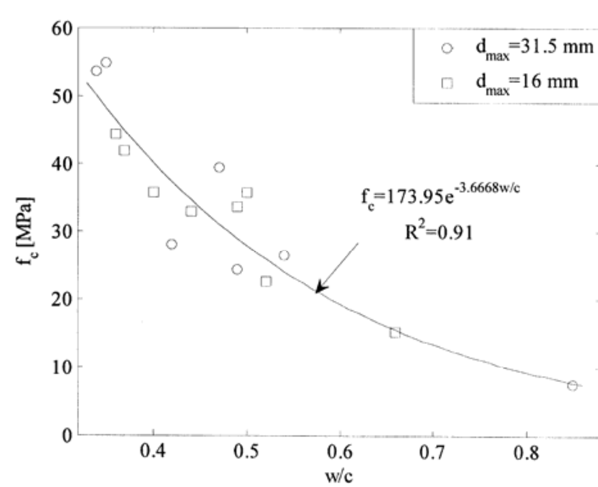

Fig. 4 — Relationship between w/c and f_c'

$\ln f_c = \ln K_1 - (w/c)\ln K_2$. Table 4 gives the results of the two-way ANOVA analysis using a significance level of 0.05 for K_{Ic}^s and $CTOD_c$. In Table 4, *df* is the degrees of freedom, *SS* is the sum of squares, *MS* is the mean squares, *F* is the *F* values and *p* is the *p*-value. The ANOVA tests results indicated that there was no change in parameters of K_{Ic}^s (p=0.410>0.05) and $CTOD_c$ (p=0.704>0.05) because of d_{max}. Besides there is a change in parameters of K_{Ic}^s (p=0.0004<0.05) and $CTOD_c$ (p=0.004<0.05) owing to w/c according to these ANOVA test results. This means that in the significant level of 5%, d_{max} differentiation on the fracture parameters is not significant, while w/c differentiation is highly significant. These results were consistent with Fig. 2.

Table 3 — Data set for ANOVA tests

Batch	w/c	d_{max}	ln (K_{Ic}^s)	ln ($CTOD_c$)
C1	1	1	3.851	3.188
C2	1	1	3.900	3.231
C3	2	1	3.290	2.973
C4	2	1	3.442	2.947
C5	2	1	3.465	3.154
C6	2	1	3.172	2.947
C7	3	1	1.992	2.347
C8	1	2	3.712	3.154
C9	1	2	3.701	3.171
C10	2	2	3.641	3.162
C11	2	2	3.340	2.916
C12	2	2	3.525	2.989
C13	2	2	3.572	3.160
C14	2	2	2.965	2.678
C15	3	2	2.899	2.795

Table 4 — Results of ANOVA

Parameter	Source	*df*	*SS*	*MS*	*F*	*p*
	w/c	2	2.41761	1.20880	17.67	0.0004
K_{Ic}^s	d_{max}	1	0.05013	0.05013	0.73	0.410
	Error	11	0.75265	0.06842		
	w/c	2	0.50511	0.25255	9.51	0.004
$CTOD_c$	d_{max}	1	0.00405	0.00405	0.15	0.704
	Error	11	0.29197	0.02654		

Figures 5 and 6 indicate the result of the regression analysis based on Abrams' Law for fracture parameters K_{Ic}^s and $CTOD_c$ in Table 2. According to the analysis results, the relationships between fracture parameters of TPM and water-cement ratio can be determined by

$$K_{Ic}^s = \frac{142}{9.6^{1.5w/c}} \quad \ldots(16)$$

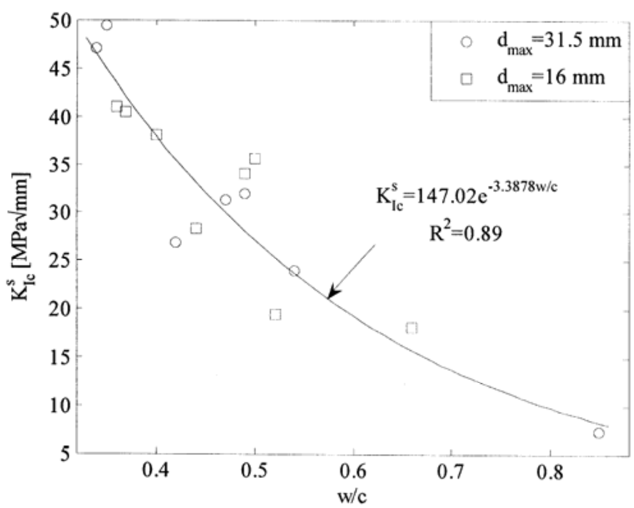

Fig. 5 — Relationship between w/c and K_{Ic}^s

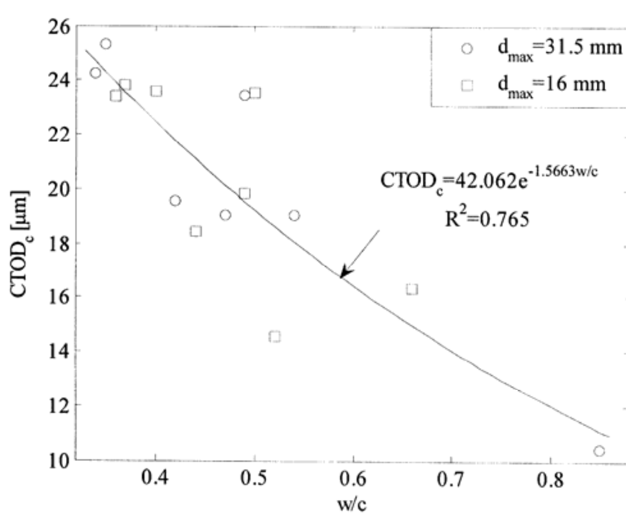

Fig. 6 — Relationship between w/c and $CTOD_c$

$$CTOD_c = \frac{42}{2.8^{1.5w/c}} \quad \ldots(17)$$

in which K_{Ic}^s and $CTOD_c$ are in MPa√mm and μm. The correlation coefficients of Eqs (16) and (17) are $r = 0.943$ and $r = 0.875$, respectively. If there are strong correlations between w/c and fracture parameters, there will naturally be good correlations between f_c' and fracture parameters. Similar to Eqs (8) and (9), the dependency of K_{Ic}^s and $CTOD_c$ on strength can be derived by using power regression based on the data in Table 2 and stated as Eqs (18) and (19) below:

$$K_{Ic}^s = 1.338\left(f_c'\right)^{0.90} \quad \ldots(18)$$

$$CTOD_c = 4.885\left(f_c'\right)^{0.41} \quad \ldots(19)$$

where f_c', K_{Ic}^s and $CTOD_c$ are in MPa, MPa√mm and μm. The correlation coefficients of Eqs (18) and (19) are $r = 0.968$ and $r = 0.885$, respectively. Eqs (8) and (9) derived by John and Shah[10] was based on very limited test data. In addition, Eq. (9) is controversial for many applications according to experimental results[18]. On the other hand, when the Eqs (18) and (8) are compared, it may be seen that Eq. (18) proposes more consistent results than Eq. (8).

Conclusions

From these experimental and statistical investigations the following conclusions can be drawn:

(i) The present experimental data indicate that the fracture parameter of two-parameter model decreases as the water to cement ratios increases. Consequently, the present test results are in good agreement with Abrams' Law. Furthermore, the regression analysis results indicate that the water-cement ratio explains about 89% of the variability of the critical stress intensity factor parameter and about 77% of variability of the critical crack tip opening displacement parameter. The reason for the good agreement with Abrams' Law is the usage of same type cement and aggregate materials and, curing and testing all the specimens under same conditions.

(ii) The proposed formulas are only developed for the rounded aggregate type and Portland cement, which had a 28 day compressive strength of 32.5 MPa. When different strength cement is used, the rates can be easily adapted by considering Eqs (18) and (19). On the other hand, if crushed or angular aggregates are used in mix design, the recent investigations by Bazant and Becq-Giraudon[12] and by the authors[22] revealed that the fracture parameters of the two-parameter model need to be increased by about 20% for the critical stress intensity factor and about 25% for the critical crack tip opening displacement.

(iii) The proposed formulas are valid only for the design of concrete with coarse aggregates. However, it is also possible to develop similar formulas for mortar.

(iv) In this study, only the fracture parameters of two-parameters model have been analyzed by using regression analysis. However, the results of this analysis can be easily adapted to the other fracture models of concrete such as the size effect model and the effective crack model, which also propose two-parameters for modelling concrete fracture.

References

1 Kaplan M F, *ACI J*, 58 (1961) 591.

2 Kesler C E, Naus D J & Lott L L, *Soc Mater Sci*, 4 (1972) 113.

3 Hillerborg A, Modeer M & Petersson P E, *Cem Conc Res*, 6 (1976) 773.

4 Bazant Z P & Oh B H, *Mater Struct (RILEM)*, 16 (1983) 155.

5 Jenq Y S & Shah S P, *ASCE J Eng Mech*, 111 (1985) 1227.

6 Nallathambi P & Karihaloo B L, *Mag Conc Res*, 38 (1986) 67.

7 Bazant Z P & Kazemi M T, *Int J Fract*, 44 (1990) 111.

8 Tang T, Ouyang C & Shah S P, *ACI Mater J*, 93 (1996) 147.

9 Tang T, Yang S & Zollinger D G, *ACI Mater J*, 96 (1999) 3.

10 John Y S & Shah S P, *ASCE J Mater Civil Eng*, 1 (1989) 185.

11 Hilsdorf H H & Brameshuber W, *Int J Fract*, 51 (1991) 61.

12 Bazant Z P & Becq-Giraudon E, *Cem Conc Res*, 32 (2002) 529.

13 Abrams D A, *Lewis Inst Bull*, 1 (1918).

14 Tada H, Paris P C & Irwin G R, *Stress analysis of cracks handbook* (St. Louis: Paris Productions), 1987.

15 Yang S, Tang T, Zollinger D G & Gurjar A, *Adv Cem Based Mater*, 5 (1997) 18.

16 RILEM Recommendation, Determination of the fracture parameters (K_{Ic}^{s} and $CTOD_c$) of plain concrete using three-point bend tests, *Mater Struct,* 23 (1990).

17 Karihaloo B L & Nallathambi P, in *Fracture Mechanics Test Methods for Concrete*, edited by Shah S P & Carpinteri A (RILEM Chapman & Hall), 1991, 1.

18 Ince R, *Eng Frac Mech*, 71 (2004) 2143.

19 Bharatkumar B H *et al*., *Mater Struct*, 38 (2005) 63.

20 ACI-318, *Building Code Requirements for Reinforced Concrete*, Detroit, 1989.

21 Pann K S, Yen T, Tang C W & Lin T D, *ACI Mater J*, 100 (2003) 311.

22 Ince R & Alyamaç K E, *Effect of aggregate type on fracture parameters of concrete*, presented at the 17th Turkish Chambers of Civil Engineers Technical Congress, Istanbul, Turkey 2004. (in Turkish)