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Backward propagation of waves in solar plasma

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In order to have negative value of phase velocity, a wave moving in the backward direction should not be expressed as exp $[i(\vec{k}.\vec{r}+\omega t)]$, but as $\exp[-i(\vec{k}.\vec{r}+\omega t)]$. As the time is always increasing, this point has been argued in the present paper and has been justified with the help of some dispersion relations derived in the literature for propagation of waves in the solar plasma. It has been shown that the expression exp $[i(\vec{k}.\vec{r}+\omega t)]$ does not give physically meaningful results.

Keywords: Dispersion relation, Magnetohydrodynamics, Plasma waves

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1 Introduction

Forward and backward propagation of waves in the solar atmosphere have been discussed from time to time. The concept of propagation of waves is shown in Fig. 1.

Suppose at time t_1 and t_2, such that $t_2 > t_1$, and time is always in the increasing order, the position vectors are $\vec{r}_1$ and $\vec{r}_2$, respectively. Then, the forward propagation is shown in Fig. 1(a) and the backward propagation in Fig. 1(b). Here, the photosphere corresponds to the origin ($\vec{r}=0$) of the coordinate system.

In order to have positive value of phase velocity, a wave moving in the forward direction may be expressed as exp $[\pm i(\vec{k}.\vec{r}-\omega t)]$. Since, the time always keeps on increasing, the forward wave is represented by the positive sign only. In the context of solar atmosphere where propagation of various types of waves is considered, a wave moving away from the photosphere is taken as in the forward direction. It is not necessary that the waves will move only away from the photosphere. They can move towards the photosphere as well. As per the convention, the direction away from the solar surface is taken as the forward direction whereas that towards the solar surface as the backward direction[1,2]. Such motion of wave is known as the backward propagation of wave.

For a perfectly conducting, compressible and homogeneous plasma in a uniform magnetic field, there exists three types of waves: (i) Alfven waves, (ii) fast-mode waves, and (iii) slow-mode waves[3]. The fast-mode waves can carry the energy even across the magnetic field and therefore, they were thought to be a carrier of energy from the photosphere to the coronal region. But the most disadvantageous characteristics of fast-mode wave is very high refraction, so that they can be reflected totally before going to the coronal region. The reflected waves propagate in the backward direction. Further, in the solar atmosphere one can consider the generation of waves by turbulent motion at magnetic reconnection site[4,5]. The generated waves can travel towards photosphere, *i.e.* in the backward direction.

For having a negative value of phase velocity, a wave moving in the backward direction may be expressed as $\exp[\pm i(\vec{k}.\vec{r}+\omega t)]$. Since the time always keeps on increasing, the backward wave should be represented by the negative sign only. Singh *et al.*[6] used the expression exp $[i(\vec{k}.\vec{r}+\omega t)]$ and such expression for a wave may be found in other studies also. This expression is not physically acceptable. Hence, the acceptable expressions for the waves moving in the forward and backward directions are exp $[i(\vec{k}.\vec{r}-\omega t)]$ and exp $[i(-\vec{k}.\vec{r}-\omega t)]$, respectively. Thus, one can always have $-\omega t$, as the time always keeps on increasing. Obviously, it is the sign of $\vec{k}$ which decides about the forward and backward propagation of the wave.

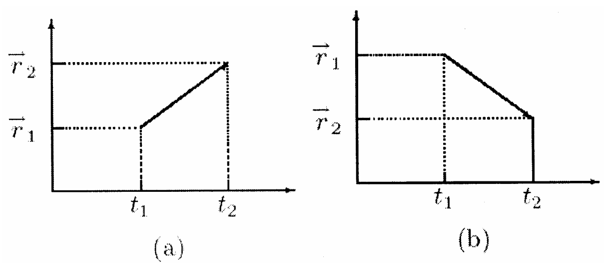

Fig. 1 — (a) Forward propagation, and (b) backward propagation - for the position vectors $\vec{r}_1$ and $\vec{r}_2$ at times t_1 and t_2

2 Discussions

The above expressions for the waves moving in the forward and backward directions can be justified with the help of dispersion relations. It is a well established fact that the dispersion relation for a wave should not depend on the direction of propagation. All the dispersion relations for the solar atmosphere considered here are derived using $\exp[i(\vec{k}.\vec{r}-\omega t)]$ for perturbation part in the process of linearization of basic equations. In these dispersion relations, one can notice that ω has even as well as odd powers, but k has only even powers. Hence, the sign of k does not matter. That is, the dispersion relation does not depend on the direction of propagation of the wave. However, the sign of ω would effects the results. For the real value of ω, the roots k_r+ik_i are obtained. The real and imaginary parts of the root are related to the wavelength, λ, and damping length, *D,* as

$$\lambda=\frac{2\pi}{\kappa_r}\text{; and } D=\frac{2\pi}{\kappa_i} \qquad \ldots (1)$$

Obviously, for a physically meaningful solution, both the k_r and k_i must be positive numbers, simultaneously.

2.1 Dispersion relation of Kumar *et al.* and Chandra & Kumthekar

The dispersion relation of Kumar *et al.*[4] and Chandra & Kumthekar[7] can be expressed as:

$$Ak^6+Bk^4+Ck^2+D=0 \qquad \ldots (2)$$

where,

$$A=\frac{3c_0\eta_0(1-\mu^2)\mu^4\omega}{\rho_0^2}+\frac{4c_0\eta_0 v_A^2\mu^4\omega}{3\rho_0 p_0}+\frac{ic_0 v_A^2\mu^4}{\rho_0}$$

$$B=v_A^2c_s^2\mu^2\omega-\frac{c_0\eta_0(1+3\mu^2)\mu^2\omega^3}{3\rho_0 p_0}+i\left[-\frac{3\eta_0 c_s^2(1-\mu^2)\mu^2\omega^2}{\rho_0}-\frac{4\eta_0 v_A^2\mu^2\omega^2}{3\rho_0}-\frac{c_0\mu^2\omega^2}{\rho_0}-\frac{c_0 v_A^2\mu^2\omega^2}{p_0}\right]$$

$$C=-(v_A^2+c_s^2)\omega^3+i\left[\frac{\eta_0(1+3\mu^2)\omega^4}{3\rho_0}+\frac{c_0\mu^2\omega^4}{p_0}\right];$$

$D=\omega^5$; and

$$\mu=\cos\theta,\qquad c_0=(\gamma-1)k_{\parallel}T_0,\qquad v_A=\frac{B_0}{\sqrt{4\pi\rho_0}},$$

$$\eta_0=10^{-16}T_0^{5/2},\qquad c_s^2=\gamma p_0/\rho_0$$

This dispersion relation remains the same when the perturbation parts of physical quantities are taken of the form $\exp[\pm i(\vec{k}.\vec{r}+\omega t)]$. For the given value of physical parameters and considering ω to be a real number and the roots of this equation are of the form $\pm(k_{r1}+i\,k_{i1})$, $\pm(k_{r2}+i\,k_{i2})$ and $\pm(k_{r3}+i\,k_{i3})$. Here, k_{r1}, k_{r2}, k_{r3}, k_{i1}, k_{i2}, k_{i3} all are positive real numbers. Obviously, for three roots, both the real and imaginary parts are positive, simultaneously. For other three roots, both the real and imaginary parts are negative.

When ω is changed to $-\omega$, the roots obtained are of the form $\pm(k_{r1}+i\,k_{i1})$, $\pm(k_{r2}+i\,k_{i2})$ and $\pm(k_{r3}+i\,k_{i3})$. Since here both the imaginary and real parts are never positive simultaneously, these roots are not physically acceptable.

Let the results obtained for the dispersion relation expressed by Eq. (2) be analyzed. One can note that in each of the coefficients *A, B, C* and *D*, the real part has odd degrees of ω, whereas the imaginary part has even degrees of ω. Let for the +ve value of ω, these are expressed as $A=a_1+ib_1$, $B=a_2+ib_2$, $C=a_3+ib_3$, $D=a_4+ib_4$, where, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 all are positive real numbers. Using these values and considering the wave vector $k=k_r+ik_i$, where k_r and k_i are real positive numbers, Eq. (2) can be written in the form of following two expression corresponding to its real and imaginary components:

$$a_1k_r^6 - 6b_1k_r^5k_i - 15a_1k_r^4k_i^2 + a_2k_r^4 + 20b_1k_r^3k_i^3$$
$$-4b_2k_r^3k_i + 15a_1k_r^2k_i^4 - 6a_2k_r^2k_i^2$$
$$+a_3k_r^2 - 6b_1k_rk_i^5 + 4b_2k_rk_i^3 - 2b_3k_rk_i$$
$$-a_1k_i^6 + a_2k_i^4 - a_3k_i^2 + a_4 = 0 \quad \ldots (3)$$

and

$$b_1k_r^6 + 6a_1k_r^5k_i - 15b_1k_r^4k_i^2 + b_2k_r^4 - 20a_1k_r^3k_i^3$$
$$+4a_2k_r^3k_i + 15b_1k_r^2k_i^4 - 6b_2k_r^2k_i^2$$
$$+b_3k_r^2 + 6a_1k_rk_i^5 - 4a_2k_rk_i^3 + 2a_3k_rk_i$$
$$-b_1k_i^6 + b_2k_i^4 - b_3k_i^2 + b_4 = 0 \quad \ldots (4)$$

Now consider −ve value of ω so that $A = -a_1 + ib_1$, $B = -a_2 + ib_2$, $C = -a_3 + ib_3$ and $D = -a_4 + ib_4$. Using these values of coefficients and taking $k = k_r - ik_i$, the dispersion relation (2) again can be expressed in form of Eqs (3) and (4).

It shows that when for the positive value of ω, the root is $k = k_r + ik_i$, then for the negative value of ω, the corresponding root is $k = k_r - ik$. Thus the +ve values of ω gives physically meaningful results as both the real and imaginary parts of k are positive. On the other side, the negative value of ω does not gives physically meaningful results as now both the real and imaginary parts are not positive.

2.2 Dispersion relation of Pekunlu *et al*.

The dispersion relation derived by Pekunlu *et al.*[8] is

$$\nu\eta k^4 + [v_A^2 - i\omega(\nu + \eta)]k^2 - \omega^2 = 0 \quad \ldots (5)$$

where, η, is the magnetic diffusivity; and ν, the coefficient of viscosity. This dispersion relation also remains the same when perturbation parts of physical quantities are taken of the form $\exp[\pm i(\vec{k}.\vec{r} - \omega t)]$. For the given value of physical parameters and considering ω to be a real number, the roots of this equation are $\pm(k_{r1} + ik_{i1})$ and $\pm(k_{r2} + ik_{i2})$. Here, obviously, for two roots, both the real and imaginary parts are positive, simultaneously. One of these roots (whose real part is larger) corresponds to the fast magnetostatic wave and the other to the slow magnetostatic wave. For other two roots, both the real and imaginary parts are negative.

When ω is changed into $-\omega$, roots of Eq. (5) are of the form of $\pm(k_{r1} - ik_{i1})$ and $\pm(k_{r2} - ik_{i2})$. Since here both the imaginary and real parts are never positive simultaneously, these roots are not physically acceptable. Thus, the expression $\exp[\pm i(\vec{k}.\vec{r} + \omega t)]$ does not give meaningful results.

3 Conclusion

It is obvious that the backward propagation of a wave can be expressed as exp $[i(-\vec{k}.\vec{r} - \omega t)]$, and not by $\exp[i(\vec{k}.\vec{r} + \omega t)]$ though both the expressions give a negative value for the phase velocity. This statement has been supported with the help of dispersion relations, where ω has both even and odd powers. Moreover, one should always have $-\omega t$, as the time goes on increasing.

Identification of such backward propagating wave is important as these waves are also the negative energy wave[9] that may introduce new instabilities. Such instabilities may play dominant role in amplification of wave and in the excitation of MHD turbulence in the solar wind[2].

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